MAC 2312 | Dec 9, 2014
Final Exam | Prof. S. Hudson

1) (15pts) Compute and simplify. Use limits on any improper integrals.

$$\frac{d}{dx}\int_0^{x^2} \sin(t^2)\ dt$$

$$\int_0^{\pi/4} \tan^2(x)\ dx$$

$$\int_0^2 |x^2-1|\ dx$$

2) (10pts) Answer True or False.

The alternating harmonic series converges conditionally.

The average value of $\cos(4x)$ on $[0, 2\pi]$ is zero.

If $|r| < 1$ then $r^2 + r^3 + r^4 + \ldots$ converges to $\frac{r^2}{1-r}$.

$\lim_{x\to\infty}(1+\frac{1}{x})^{3x} = e^3$

The polar equations $r = \sin 2\theta$ and $r = -\sin 2\theta$ have the same graphs.

3) (10pts) Suppose a particle has position $s(t)$ after t seconds. Suppose the acceleration is $a(t) = 2 + t$, the initial velocity is $v_0 = 3$ and the initial position is $s(0) = s_0 = 4$. Find the displacement for the period $0 \le t \le 2$.

4) (10pts) Find the radius and interval of convergence of $\sum_{k=1}^{\infty} \frac{5^k}{k^2} x^k$.

5) (5pts) Classify the series (as D, CA or CC) with a brief justification.

$\sum_{k=1}^{\infty} \frac{(-1)^k k+2}{k(k+3)}$

6) (5pts) Classify the series (as D, CA or CC) with a brief justification.

$\sum_{k=1}^{\infty} \frac{\sin(k)}{k^3}$

7) (10pts) Approximate $\tan^{-1}(0.1)$ to three decimal places using a McLaurin Series. You do not have to simplify any fractions/roots/etc. Explain your method briefly (how you made the best possible choice of n).

8) (10pts) Find the first five nonzero terms of the McLaurin Series for $f(x) = \frac{4x-2}{x^2-1}$. For full credit, use partial fractions and 1-2 known series.

9) (7 pts) What kind of curve is $r = 2 + 2\sin(\theta)$ (name it)? Find dy/dx at the point where $\theta = 0$.

10) (8 pts) Find the area inside the curve $r = 2\cos(3\theta)$.

11) (10 pts) Choose ONE to do (as usual, you can continue on the back);

A. State and prove the Divergence Test.

B. State and prove the Comparison Test.

C. Prove that the Harmonic Series diverges directly from the definition.

Bonus) (3 pts) According to a recent article, who was the meanest man in math ?

Remarks and Answers: The average score was 48 based on the top 18 scores, which is unusually low. The high score was 68, followed by many scores in the 50's. The average score varied from about 40% to 65% on most of the problems, with a low of 22% on Problem 7 (though this was assigned as HW, see 9.9.6). I do not set a separate scale for finals, and have not yet set one for the semester grade.

1a) $2x\sin(x^4)$. Use the FTC and Chain Rule: $\frac{d}{dx}A(x^2) = A'(x^2)\ 2x = \sin(t^2)|_{t=x^2}\ 2x$.

1b) $1 - \pi/4$, from $\int \tan^2 x = \int(\sec^2 x - 1) = \tan(x) - x$.

1c) Use the abc thm (with $b = 1$) to get $2/3 + 4/3 = 2$.

2) TTTTT

3) 34/3, from $v = \int a = t^2/2 + 2t + 3$, $s = \int v = t^3/6 + t^2 + 3t + 4$, $s(2) - s(0) = 34/3$.

4) The RTAC gives $\rho = 5|x| < 1$, so $|x| < 1/5$, $R = 1/5$, $I = [-1/5, 1/5]$ (after checking that both endpoints are included).

5) CC [3 points for this] . It converges by the A.S.Test, but fails to CA by the Limit Comparison Test (with $1/k$) [2 points for this - but other explanations are possible].

6) CA, by comparing $|\sin(k)|/k^3$ with $1/k^3$. I think that any other good explanation must at least include the absolute values.

7) Since we know (from the usual table) that $\tan^{-1}(x) = x - x^3/3 + x^5/5 \ldots$ for $|x| < 1$, we can set $x = 0.1$, and express $\tan^{-1}(0.1)$ as the sum S of an alternating series. The term $(0.1)^3/3$ is already less than 0.0005 so we can approximate S by just the first term, 0.1.

We might try to use the usual bound on $R_n(0.1)$ instead (and this idea usually got partial credit). But that does not work out very well because M is rather hard to compute for this function (sorry - didn't notice this issue in time).

8) From PFs, $f = \frac{1}{x-1} + \frac{3}{x+1}$. We all know the series $\frac{1}{1-x} = 1 + x + x^2 + \ldots$ and $\frac{1}{1+x} = 1 - x + x^2 - \ldots$. Multiply these by -1 and by 3, then add, to get

$$f(x) = 2 - 4x + 2x^2 - 4x^3 + 2x^4 \ldots$$

Plan 2: We can get the series for $\frac{1}{x^2-1}$ and multiply that by $4x - 2$ (try it!).

Plan 3: We could use the usual $c_n = \frac{f^{(n)}(0)}{n!}$, except that this gets messy for large n (and violates the instructions to use PFs).

9) Cardioid (I counted "limacon" as OK, but not quite as good). $dy/dx = 2/2 = 1$.

10) This is a 3-petal rose; a graph is highly recommended to set-up the calculation, but I did not require it. $A = \frac{1}{2}\int_0^{\pi} 4\cos^2(3\theta)d\theta = \pi$. I used $\beta = \pi$ here, because that's when retracing starts. I also took a shortcut to compute the integral, replacing $\cos^2(3\theta)$ by $1/2$. But you should check that, and not use it unless you understand when it works.

It is also OK (and maybe safer) to use symmetry. The area is 6 times that of half a petal: $A = \frac{6}{2}\int_0^{\pi/6} 4\cos^2(3\theta)d\theta = \pi$. Here, I got $\beta = \pi/6$ by setting $\cos(3\beta) = 0$. Remark: Very few people got this exactly right. If you want partial credit on a problem like this, at least explain your plan, including your choice of β, probably also including a graph.

11) See the text. For each, you should include *partial sums*.

Bonus: J.Bernoulli.