MAC 2312 April 25, 2017
Final Exam Prof. S. Hudson

1) (20pts) Compute and simplify. Use limits on any improper integrals.

$$\frac{d}{dx}\int_0^{x^2} \sin(t^2)\ dt$$

$$\int x\ln(x)\ dx$$

$$\int_0^2 |x^2-1|\ dx$$

$$\int \frac{2x^2+3x-2}{x^3-x^2-2x}\ dx$$

2) (10pts) Find an integral for the volume of the solid obtained by revolving the region bounded by $x=0$, $x+y=4$ and $y=0$ around the line $x=\ -1$. Shells are suggested but other methods are OK.

3) (10pts) Answer True or False.

If $\lim a_k = 0$ then $\sum a_k$ converges.

The average value of $\cos^2(4x)$ on $[0,2\pi]$ is $1/2$.

The Maclaurin series for $\tan^{-1}x$ (centered at $x_0$) converges at $x=\pi$.

The polar equations $r=\sin 3\theta$ and $r=-\sin 3\theta$ have the same graphs.

No Riemann sum for $\int_0^5 \sin(x)\ dx$ can exceed 5.

4) (10pts) Suppose a particle has position $s(t)$ after $t$ seconds. Suppose the acceleration is $a(t)=t$, the initial velocity is $v_0=3$ and the initial position is $s(0)=s_0=4$. Find the position of the particle after 3 seconds.

5) (10pts) Find the radius and interval of convergence of $\sum_{k=1}^{\infty}\frac{3^k x^k}{k+2}$.

6) (5pts each) Classify each series (as D, CA or CC) with a brief justification.

6a) $\sum_{k=2}^{\infty}\frac{k^3}{k^4-1}$

6b) $\sum_{k=2}^{\infty}(\frac{-1}{\ln k})^k$

7) (8 pts) Find the Maclaurin series for $f(x) = \sqrt{1+3x}$. Write out at least 3 terms. For full credit, write your answer in sigma notation.

8) (10 pts) Sketch the curve $r = 2 + 2\cos(\theta)$. Find the area of the region inside it.

9) (7 pts) Approximate ln(2). Use any one of the following methods. You can leave your answer as a short sum of fractions.

a) Write this as an integral (using the definition of ln(x)) and approximate that by $T_2$.

b) Same as (a) but use the Left Endpoint Rule with $n = 3$ and $x_1 = 1.5$ and $x_2 = 1.75$.

c) Use a partial sum of a well-known series, and get an error less than 0.2.

10) (5 pts) Read the bonus and then choose ONE. State it carefully, including all hypotheses.

A. State the Divergence Test.

B. State the Comparison Test.

C. State either version of the Fundamental Theorem of Calculus.

Bonus) (5 pts) Prove the theorem you chose in the previous problem.

**Remarks and Answers:** The average was 50 / 100, which is low, with high scores of 94 and 72. The lowest scores were on problem 9 (25%) and perhaps 1a, 1c. The best were on problem 10 (74%). As usual, I do not directly scale the final exam, and have not yet curved the semester grades.

1a) $2x\sin(x^4)$

1b) $x^2(2\ln x - 1)/4 \ + C$

1c) 2

1d) $\ln x + 2\ln(x-2) - \ln(x+1) + C$

2) $\int_0^4 2\pi(x+1)(4-x)\ dx$

3) FTFFT

4) 35/2.

5) $R = 1/3$, $I = [-1/3, 1/3)$.

6a) D, using the Comp Test (but other tests may work too).

6b) CA, using the Root Test.

7) $1 + (\frac{1}{2})3x + (\frac{1}{2})(\frac{-1}{2})\frac{9x^2}{2!} \cdots$ For the sigma notation, see the text (set m=1/2 and sub-in 3x).

8) $6\pi$.

9a) $T_2 = \frac{1}{4}[1 + \frac{4}{3} + \frac{1}{2}]$

9b) $\frac{1}{2} + \frac{1}{4} \cdot \frac{2}{3} + \frac{1}{4} \cdot \frac{4}{7}$

9c) $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4}$.

10, and the bonus) See the text or lecture notes.