MAC 2312 [noon] Apr 24, 2018
Final Exam + Key Prof. S. Hudson

1) (10pts) Find $\sum_{k=1}^{n} f(x_k^*)\Delta_{x_k}$ given that $f(x) = x + 1$, $a = 0$, $b = 4$, $n = 3$,
$\Delta_{x_1} = 1$, $\Delta_{x_2} = 1$ and $\Delta_{x_3} = 2$ and
$x_1^* = 1/3$, $x_2^* = 3/2$, $x_3^* = 3$.

2) (10pts) Find an integral for the volume of the solid obtained by revolving the region bounded by $x = 0$, $x + y = 3$ and $y = 0$ around the line $x = -2$. Shells are suggested, but other methods are OK. You do not have to evaluate the integral.

3a) (5pts each) Compute $\int x \ln(3x)\, dx$

3b) Compute $\int \frac{x^2+2}{x^3-2x^2-3x}\, dx$

4) (5pts each) Classify each series as D for divergent, CA for absolutely convergent, or CC for conditionally convergent. Justify your answers with convergence tests and calculations.

$\sum_{k=2}^{+\infty} \frac{k^2}{k^3-1}$

$\sum_{k=2}^{+\infty} \frac{(-1)^{k+1}}{\ln(k)}$

5) (10 pts) Find the McLaurin series for $f(x) = \sqrt{1 + 5x}$. Write out at least 3 terms. For maximum credit, also write your answer in Sigma notation.

6) (10pts) Find the interval of convergence of $\sum_{k=2}^{\infty} \frac{(3x)^k}{5k}$.

7) (10pts) Find the area of the region inside the circle $r = 2\sin\theta$ but outside the circle $r = 1$. Small hint #1: if you use the same notation I do, you can set $\alpha = \pi/6$, but compute β yourself. Small hint #2: I said in class that a term like $\cos(2\theta)$ often arises in problems like this one, but often has no effect on the final answer. You should be more careful here.

8) (20pts) Answer True or False:

The curve $r = 5\cos(3\theta)$ has at least 5 different horizontal tangent lines.

$\lim_{x\to\infty}(1 + \frac{3}{x})^x = e^3$

The Maclaurin series for $\tan^{-1}(x)$ converges for all x.

If $0 \le a_n \le b_n$ and $\lim b_n$ converges, then $\lim a_n$ converges.

The area enclosed by $r = 3\sin(2\theta)$ is equal to the area enclosed by $r = 3\cos(2\theta)$.

$\lim_{n\to\infty} \cos(2n\pi) = 1$.

Every monotonic sequence that is bounded above converges.

If $a_0 = 3$ and $a_{n+1} = 5 - a_n, \forall n \ge 1$, then $a_{2018} = a_{1940}$.

The alternating harmonic series has bounded partial sums.

The average value of $\ln(x)$ on $[1/2, 2]$ is positive.

9) (10pts) Choose ONE to do;

A. State and prove the integral formula for area in polar coordinates.

B. State and prove the Divergence Test.

Remarks and Answers: The average was approx 56, which a bit low. The best scores were 76 and 73. The average results on each problem were similar, except maybe problem 4 (below 40%). I do not set a separate scale for the final, but will include this when setting the semester scale.

1) $(1/3\ +1)\cdot 1 + (3/2\ +1)\cdot 1 + (3+1)\cdot 2 = 71/6$.

This is an early problem from approx Ch.5.4 or 5.5. It might help to imagine this is the area of three rectangles, added. A fairly common mistake was to use 1/3 instead of $(1/3\ +1)$, etc. Maybe ignoring that $f(x) = x + 1$?

2) $V = \int_0^3 2\pi(x+2)(3-x)\ dx$. The radius is $x - (-2)$ and the height is $y = x - 3$. A picture is strongly suggested.

3a) Using IBP, $\frac{x^2 \ln(3x)}{2} - \frac{x^2}{4} + C$. Notice that $\frac{d}{dx}\ln(3x) = \frac{1}{x}$ by the Chain Rule or from $\ln(3x) = \ln(3) + \ln(x)$.

3b) $(-2/3)\ln|x| + (11/12)\ln|x-3| + (3/4)\ln|x+1| + C$. Notice that $Q(x) = x(x-3)(x+1)$. You can get $A = -2/3$ etc, using the shortcut, or using the standard method.

4a) D. Compare with $\sum 1/k$.

4b) CC. Use the AST (for C) and compare with $\sum 1/k$ (vs CA). Note $\ln(k+1) < k$.

5) You can substitute $5x$ into the usual binomial formula (in Table 9.9.1) and get $1 + (1/2)5x + (1/2)(-1/2)(5x)^2/2 + \cdots$. You can simplify it to $1 + 5x/2 - 25x^2/8 + \cdots$

For full credit, write something like $1 + \sum_{k=1}^{\infty}[-1\cdot 1\cdot 3\cdot 5\cdots(2k-3)]\frac{(-5x/2)^k}{k!}$. There are various ways to write it (but none are very pretty, as far as I know).

6) Using RTAC with $\rho = \cdots = |3x| < 1$, we find the endpoints of I are $\pm 1/3$. After checking these separately, as usual (but many people skipped this step), we get $I = [-1/3, 1/3)$. You can factor out the 5 from the start, if you like, and just ignore it.

7) Draw the circles, which overlap, and perhaps two special radii. Subtract two areas. $A = \frac{1}{2}\int_{\pi/6}^{5\pi/6}(2\sin\theta)^2 - 1^2\ d\theta = \cdots = \pi/3 + \sqrt{3}/8$. It is also OK to use symmetry and start

with $A = \int_{\pi/6}^{\pi/2}$ [etc]. The most common problem by far was a bad start.

8) FTFFT TFTTT

9) See the textbook.