MAC 2312 [2pm] Apr 26, 2018
Final Exam and Key Prof. S. Hudson

1) (10pts) A particle moves along the $s$-axis. Given that $v(t) = 3t^2 - 2t$ and $s(0) = 1$, find the position function of the particle.

2) (10pts) Find an integral for the volume of the solid obtained by revolving the region bounded by $x = 0$, $y = 5 - x$ and $y = 0$ around the line $x = -1$. Shells are suggested, but other methods are OK. You do not have to evaluate the integral.

3a) (5pts each) Compute $\int 2x \ln(x)\ dx$

3b) Compute $\int \frac{x^2+3x-2}{x^3+2x^2-3x}\ dx$

4) (5pts each) Classify each series as D for divergent, CA for absolutely convergent, or CC for conditionally convergent. Justify your answers with convergence tests and calculations.

$\sum_{k=2}^{+\infty} \frac{k^4}{k^5-1}$

$\sum_{k=2}^{+\infty} \frac{(-1)^k}{\ln(k+1)}$

5) (10 pts) Find the McLaurin series for $f(x) = \sqrt{1 + 2x}$. Write out at least 3 terms. For maximum credit, also write your answer in Sigma notation.

6) (10pts total) a) Write Lagrange's bound on $R_n$ (the one with $M/(n+1)!$ in it).

b) Write the McLaurin series for $\sin(x)$ (from memory is OK).

c) Use the bound in part a) to show this series converges to $\sin(x)$ for all $x$ (show that the remainder goes to 0).

7) (10pts) Find the area of the region inside the cardioid $r = 1 + \cos(\theta)$.

8) (20pts) Answer True or False:

The series $2 - 1 - 1 + 2 - 1 - 1 + 2 - 1 - 1 \ldots$ converges to 0.

$\lim_{n\to\infty} \cos(2n\pi) = 1$.

Every decreasing sequence that is bounded above converges.

If $a_0 = 2$ and $a_{n+1} = 5 - a_n, \forall n \geq 1$, then $a_{2018} = a_{1942}$.

The alternating harmonic series converges to $\ln(2)$.

The harmonic series has bounded partial sums.

The average value of $\cos(5x)$ on $[0, 2\pi]$ is zero.

If $|r| < 1$ then $r^2 + r^3 + r^4 + \ldots$ converges to $\frac{r^2}{1-r}$.

$\lim_{x\to\infty}(1+\frac{3}{x})^x = e^3$

The polar equations $r = \sin 2\theta$ and $r = \cos 2\theta$ have the same graphs.

9) (10pts) Choose ONE to do;

A. State and prove the integral formula for area in polar coordinates.

B. State and prove the Comparison Test.

**Remarks and Answers:** The average was approx 66, which is fairly normal, maybe even a bit high for a final. The best scores were 94 and 93. The average results on each problem were OK, except maybe problem 6 (approx 40%). I do not set a separate scale for the final, but will include this when setting the semester scale.

1) $s(t) = t^3 - t^2 + 1$.

2) $V = \int_0^5 2\pi(x+1)(5-x)\ dx$. I suggest using a sketch.

3a) $x^2\ln(x) - x^2/2 + C$

3b) $(1/2)\ln|x| + (-1)\ln|x-1| + (8/3)\ln|x+1| + C$. Notice that $Q(x) = x(x-1)(x+3)$. You can get $A = 1/2$ etc, using the shortcut, or using the standard method.

recheck this ans...

4a) D, by the Comp.Test (with $b_k \geq 1/k$). Or, use the LCT or perhaps the integral test (I have not checked that $f(x)$ decreases, but it probably works).

4b) CC. It converges by the AST but not absolutely by comparison with $1/k$ (because $\ln(k+1) < k$ eventually).

5) You can substitute $2x$ into the usual binomial formula (in Table 9.9.1) and get $1 + (1/2)2x + (1/2)(-1/2)(2x)^2/2 + \cdots$.You can simplify it to $1 + x - x^2/2 + \cdots$

For full credit, write something like $1 + \sum_{k=1}^{\infty}[-1\cdot 1\cdot 3\cdot 5\cdots(2k-3)]\frac{(-x)^k}{k!}$. There are various ways to write it (but none are very pretty, as far as I know).

6a) $R_n(x) \leq \frac{M|x-x_0|^{n+1}}{(n+1)!}$. 6b) $x - x^3/3! + x^5/5! - \cdots$. 6c) Part 6a) simplifies to $R_n(x) \leq \frac{|x|^{n+1}}{(n+1)!}$ and we saw in class that $\lim \frac{|x|^{n+1}}{(n+1)!} = 0$ for all $x$. So, the remainder also goes to zero, which means the series converges to $\sin x$ for all $x$.

The results were low, though parts (a) and (b) were only memorization, and the results seemed OK on (b).

7) $A = \frac{1}{2}\int_0^{2\pi}(1+\cos\theta)^2\ d\theta = \cdots = 3\pi/2$. I strongly suggest a sketch and a sanity check. This is not required, but I do give more partial credit for reasonable answers than for crazy

ones (like $A = 0$ or $A = 18\pi$).

8) FTFTT FTTTF

9) See the text.