MAC 2312 April 23, 2020
Online Final Exam + Key Prof. S. Hudson

The rules again briefly: Be alone and silent. Audio on, video always on your hands. No help from books, papers, gadgets, people, etc. Cooperate with the proctor and tell him asap of any tech problems / etc.

1) (10 pts) Compute and simplify. Use limits on any improper integrals.

(a) $\displaystyle\int x^2 \sin x \; dx$

(b) $\displaystyle\int \frac{3}{x^2 - 6x + 8} \; dx$

2) (20 pts) Answer True or False.

The alternating harmonic series converges conditionally.

If every partial sum of $\sum_{k=1}^{\infty} a_k$ is less than 100, the series converges.

Simpson's rule is exact if f is a cubic polynomial.

The average value of $\sin^2(x)$ on $[0, 2\pi]$ is 1/2.

$\sum_{k=1}^{\infty} 2^{-k} = 1$ but it can be rearranged to converge to 7.

Every Riemann sum for $\int_1^5 x \; dx$ is less than 12.

$\lim_{x\to\infty}(1 + \frac{1}{x})^x = e$

The integral $\int_1^{+\infty} 1/x^p \; dx$ converges if and only if $p < 1$.

$\sum_{k=1}^{100} k = 5550$

A bounded increasing sequence must converge.

3) (10 pts) Find the volume of the solid obtained by revolving the region bounded by $x = 0$, $y = \sqrt{x}$ and $y = 2$ around the y-axis. For full credit, indicate the method you are using (disks, shells, etc) and sketch the region and the solid. Set up of the integral **and** computation are required.

4) (10 pts) Classify each series as absolutely convergent, conditionally convergent or divergent. Justify your answers.

(a) $\displaystyle\sum_{k=1}^{\infty}(-1)^k \frac{3}{3k-1}$

(b) $\displaystyle\sum_{k=0}^{\infty} \frac{(-3)^k}{k!}$

5) (15 pts) Find the Taylor series for each, showing all work.

5a) For $f(x) = e^x$ at the point $a = 2$. For full credit your answer should use summation notation.

5b) Find the Taylor Series for $f(x) = x\sin(2x)$ at $a = 0$ (this is also called a Mclaurin Series).

6) (8 pts) Estimate the error if $P_3(x) = x - (x^3/6)$ is used to estimate the value of $\sin x$ at $x = 0.1$ radians.

7) (10 pts) Find the area of the region inside the cardioid $r = 1 + \cos(\theta)$. Include a sketch of the curve.

8) (7 pts) Find the slope of the tangent line to the polar curve $r = 2\cos(\theta)$ at $\theta = \pi/3$. For full credit, use the chain rule formula for dy/dx as in class.

9) (10 pts) Choose ONE;

A. State and prove the integral formula for area in polar coordinates.

B. State and prove the Comparison Test.

Remarks and Answers: The high scores were 90 and 81 with an average of 57. The average on each problem was approx 60% to 70% except for low scores on problems 6 and 8 (approx 25% on each). I do not have a separate scale for the final exam. This final will probably not affect the semester much, though the HW may raise it a couple of points from the one on the Exam II Key.

1a) $-x^2 \cos x + 2 \sin x + 2 \cos x + C$

1b) $\frac{3}{2} \ln(|x-4|/|x-2|) + C$. Be sure to show all your work, behind $A = 3/2$, for example.

2) TFTTF FTFFT

3) Using Disks, $V = \pi \int_0^2 (y^2)^2 \, dy = \cdots = 32\pi/5$. Shells are also OK.

4a) CC. It converges by the AST, but not CA by comparison with the HS. Include some calculations.

4b) CA, by the Ratio Test. Include some calculations.

5a) Using $f^{(n)}(a) = e^2$, get $e^2 + e^2(x-2) + e^2(x-2)^2/2! + \cdots$, or $\sum_{n=0}^{\infty} e^2(x-2)^n/n!$.

5b) The $f^{(n)}(a)$ method is a little messier for this one, so work from the known series, $\sin x = x - x^3/3! + \cdots$. Substituting $2x$, get $\sin 2x = 2x - 8x^3/3! + \cdots$ and multiply both sides by x to get

$$x \sin 2x = 2x^2 - 4x^4/3 + \cdots.$$

I did not require sigma notation for 5b. I did require the calculations, especially with the $f^{(n)}(a)$ method.

6) $|R_3(0.1)| \leq M(0.1)^4/4!$, where we can set $M = 1$. So, $|R_3(0.1)| \leq \frac{1}{240000}$.

7) $A = \frac{1}{2} \int_0^{2\pi} (1 + \cos\theta)^2 \, d\theta = \cdots = 3\pi/2$. This is another example where we need the trig identity for $\cos^2\theta$.

8) $\frac{1}{\sqrt{3}}$ using the chain rule method (which appears in both 11.2 and 11.4 I think).

9) See the textbook or lecture notes.