Quiz 2 Oct 29, 2020
MAC 2312 Key S Hudson

1) [30pts] Compute and simplify. If it diverges, show some work and explain.

$\int_{-\infty}^{+\infty} \frac{4xdx}{x^2+1}$

2) [40pts] Compute and simplify.

$\int \frac{2dx}{x(3x-1)}$

3) [30pts] Answer True or False:

The partial sums of the series $\sum_{k=1}^{\infty}(-1)^k$ are bounded.

Eventually the partial sums of the series $\sum_{k=1}^{\infty} 5-k$ are monotonic.

The series $2-1-1+2-1-1+2-1-1\dots$ converges to 0.

Every monotone sequence that is bounded above converges.

If $a_0 = 1$ and $a_{n+1} = 3 - a_n, \forall n \geq 1$, then $a_{2020} = a_{2000}$.

Remarks: The average was approx 68, with two students tied for the high score of 98. This is one of the better results so far this term.

1) Diverges. $\int_{-\infty}^{+\infty} \frac{4xdx}{x^2+1} = \int_{-\infty}^{0} \frac{4xdx}{x^2+1} + \int_{0}^{+\infty} \frac{4xdx}{x^2+1}$. The rule is to split any improper integral with two 'issues'. We need to check that one diverges, $\int_{0}^{+\infty} \frac{4xdx}{x^2+1} = \lim_{M\to+\infty} \int_{0}^{M} \frac{4xdx}{x^2+1}$, and set $u = x^2+1$ to continue and finish.

Don't start with $\int_{-M}^{+M} \frac{4xdx}{x^2+1}$ which leads incorrectly to a limit of 0. This was Cauchy's idea, discussed in class.

2) $\int \frac{2dx}{x(3x-1)} = \int \frac{-2dx}{x} + \int \frac{6dx}{3x-1} = -2\ln|x| + 2\ln|3x-1| + C$. The first step is Partial Fractions and the second uses $u = 3x-1$. The answer can be rewritten in various ways, but I did not require it.

3) TTFFT. We went over these in class after the quiz.