Key

Evaluate each of these Integrals:

41. $\int \sqrt{\frac{x^3-3}{x^{11}}}\,dx$

$u = x^3 - 3 \rightarrow x^3 = u+3$

$du = 3x^2\,dx$

$\frac{du}{3x^2} = dx$

$$\Rightarrow \int \sqrt{\frac{u}{x^{11}}} \cdot \frac{du}{3x^2}$$

$$= \frac{1}{3}\int \frac{\sqrt{u}}{\sqrt{x^{11}}} \cdot \frac{du}{\sqrt{x^4}}$$

$$= \frac{1}{3}\int \frac{\sqrt{u}\cdot\sqrt{1}}{\sqrt{x^{11}}\cdot\sqrt{x^4}}\,du$$

$$= \frac{1}{3}\int \frac{\sqrt{u}}{\sqrt{x^{15}}}\,du$$

$$= \frac{1}{3}\int \frac{\sqrt{u}}{\sqrt{x^6\cdot x^6\cdot x^3}}\,du$$

$$= \frac{1}{3}\int \frac{\sqrt{u}}{\sqrt{x^6}\cdot\sqrt{x^6}\cdot\sqrt{x^3}}\,du$$

Sub in $x^3 = u+3$

$$= \frac{1}{3}\int \frac{1}{\sqrt{x^6}}\cdot\frac{1}{\sqrt{x^6}}\cdot\frac{1}{\sqrt{x^2}}\cdot\frac{\sqrt{u}}{\sqrt{x^3}}\,du \Rightarrow \frac{1}{3}\int \frac{1}{u+3}\cdot\frac{1}{u+3}\cdot\sqrt{\frac{u}{u+3}}\,du$$

$$= \frac{1}{3}\int \frac{1}{(u+3)^2}\sqrt{\frac{1}{u+3}\cdot u}\,du$$

$h = \frac{1}{u+3} \Rightarrow u = \frac{1-3h}{h}$

$dh = -(u+3)^{-2}\,du$

$-(u+3)^2\,dh = du$

$$= \frac{1}{3}\int \cancel{\frac{1}{(u+3)^2}}\sqrt{u\cdot h}\cdot\left(-\cancel{(u+3)^2}\right)dh$$

$$= -\frac{1}{3}\int \sqrt{u\cdot h}\,dh$$

$$= -\frac{1}{3}\int (1-3h)^{1/2}\,dh$$

$$= -\frac{1}{3}\left(\frac{-2}{9}\cdot(1-3h)^{3/2}\right) + C$$

$$= \frac{2}{27}\left(1-\frac{3}{u+3}\right)^{3/2} + C$$

$$= \boxed{\frac{2}{27}\left(1-\frac{3}{x^3}\right)^{3/2} + C}$$

49. $\int \frac{x}{(x^2-4)^3}\,dx$

$u = x^2 - 4$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

power rule for integrals

$\int x^n\,dx = \frac{x^{n+1}}{n+1}$

$= \int \frac{1}{2u^3}\,du$

$= \frac{1}{2}\int u^{-3}\,du$

$= \frac{1}{2}(-\frac{1}{2}u^{-3}) + C$

$= -\frac{1}{4}(x^2-4)^{-2} + C$

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31 $\int \frac{\sin(2t+1)}{\cos(2t+1)}\,dt$

$h = 2t+1$

$dh = 2dt$

$\frac{dh}{2} = dt$

note:

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sec\theta = \frac{1}{\cos\theta}$

$\Rightarrow \int \frac{\sin h}{2\cos^2 h}\,dh$

$\Rightarrow \int \frac{\tan(h)}{2\cos(h)}\,dh$

$= \int \frac{1}{2}\tan(h)\sec(h)\,dh$

$= \frac{1}{2}\sec(h) + C$

$= \frac{1}{2}\sec(2t+1) + C$

Activity 2: Answer True or False to each of these problems.

1. The average value of $f(x) = x^4$ on $[1, 7]$ is equal to $f(4)$.
2. If f is continuous on $[a, b]$, then it has an antiderivative there.
3. If f is integrable on $[-5, 0]$, then it is also differentiable there.
4. The area between the curves $y = 2x$ and $y = x^2$ is $A = \int_0^2 2x - x^2\, dx$.
5. If velocity is positive for some time interval, then the distance traveled is the displacement.

① $\text{av}f(x) = \frac{1}{7-1} \int_1^7 x^4$

$= \frac{1}{6}\left(\frac{1}{5}x^5\right]_1^7 = \frac{1}{6}\left(\frac{1}{5}(7^5 - 7)\right) = 560$

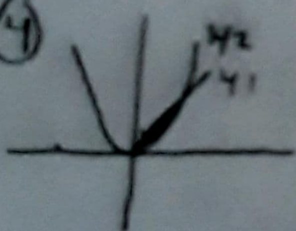
$f(4) = (4)^4 = 256$

$* \; f(4) \neq \text{av}f(x) \rightarrow$ False.

② True, definition of area under the curve.

③ False, Just because its integrable doesn't mean its differentiable.

④



$y_1 = 2x$

$y_2 = x^2$

$*$ intersect @ $x = 0, x = 2$

$A = \int_0^2 (\text{top} - \text{bottom})\, dx$

$= \int_0^2 (2x - x^2)\, dx$

True

⑤

$\text{velocity} = \frac{d}{dt}(\text{position})$

$\boxed{\text{true}}$

Activity 3:

$$\vec{v} = 6\sin(2t)\ \frac{m}{sec}$$

$s = 0,\ t = 0$

Find s when $t = \frac{\pi}{2}$ seconds.

$$\vec{V} = \frac{d\vec{s}}{dt}$$

$$\int ds = \int \vec{v}\, dt$$

$w = 2t$

$\frac{dw}{2} = dt$

$t = \pi/2 \rightarrow w = \pi$

$t = 0 \rightarrow w = 0$

$$s = \int_0^{\pi/2} 6\sin(2t)\, dt$$

$$\Rightarrow \int_0^{\pi} \frac{6}{2}\sin(w)\, dw$$

$$\Rightarrow 3\int_0^{\pi} \sin(w)\, dw$$

$$= 3\left(-\cos(w)\right]_0^{\pi}$$

$$= 3(-\cos(\pi) + \cos(0))$$

$$= 3(1 + 1)$$

$$= 6$$