12.5-13.4 HOMEWORK

Graded by Learning Assistant Enzo De Oliveira.

I numbered the problems from each section and used a random number generator to select one problem from each section for 6 problems total.

The questions graded were

- Section 12.5: 23
- Section 12.6: 11
- Section 13.1: 1
- Section 13.2: 23
- Section 13.3: 6
- Section 13.4: 11

Each question was out of 15 points, for a total of 90 points for accuracy. Another 10 points was awarded for neatness, legibility, work, stapling etc.

1. Section 12.5: 23

Find equation of the plane through $(1, 1, -1)$, $(2, 0, 2)$, and $(0, -2, 1)$

Solution: I'll label the points to not cause confusion: A(1, 1, -1), B(2, 0, 2), and C(0, -2, 1).

$$\vec{u} = \vec{AB} = (2-1)\vec{i} + (0-1)\vec{j} + (2-(-1))\vec{k} = \vec{i} - \vec{j} + 3\vec{k}$$

$$\vec{v} = \vec{BC} = (0-2)\vec{i} + (-2-0)\vec{j} + (1-2)\vec{k} = -2\vec{i} - 2\vec{j} - \vec{k}$$

Finding these vectors was worth 5 points.

Both $\vec{u}$ and $\vec{j}$ lie on the plane. We must now find the normal vector of the plane.

$$\vec{n} = \vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 3 \\ -2 & -2 & -1 \end{vmatrix} = (1+6)\vec{i} - (-1+6)\vec{j} + (-2-2)\vec{k} = 7\vec{i} - 5\vec{j} - 4\vec{k}$$

Finding the normal vector was worth 5 points.

The equation of the plane using normal vector, $\vec{n}$, and point $B(2, 0, 2)$

$$7(x-2) - 5(y-0) - 4(z-2) = 0 \Rightarrow 7x - 5y - 4z = 6$$

Finding the correct equation was worth 5 points.

2. Section 12.6: 11

Match the equation with the surface it defines. Also, identify each surface by type (paraboloid, ellipsoid, etc.). The surfaces are labeled (a)–(1).

$$x^2 + 4z^2 = y^2$$

Solution:

(h.), Cone

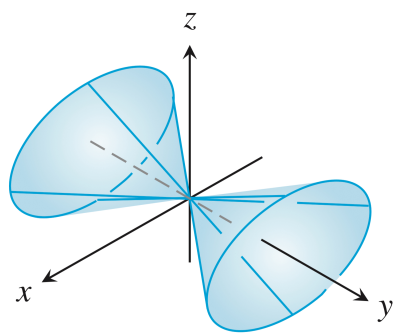

Figure 1. Graph of H

If only one of the criteria above were met, 8 points were awarded. Getting both earned 15 points.

3. Section 13.1: 1

$$\lim_{t \to \pi} \left[\left(\sin \frac{t}{2} \right) \vec{i} + \left(\cos \frac{2}{3} t \right) \vec{j} + \left(\tan \frac{5t}{4} \right) \vec{k} \right]$$

Solution:

$$\left(\sin \frac{\pi}{2} \right) \vec{i} + \left(\cos \frac{2}{3} \pi \right) \vec{j} + \left(\tan \frac{5\pi}{4} \right) \vec{k}$$

Correctly evaluating the limit was worth 10 points.

$$(1)\vec{i} - \frac{1}{2}\vec{j} + (1)\vec{k} = \vec{i} - \frac{1}{2}\vec{j} + \vec{k}$$

Getting the correct final answer was awarded 5 points.

4. Section 13.2: 23

Projectile flights in the following exercises are to be treated as ideal unless stated otherwise. All launch angles are assumed to be measured from the horizontal. All projectiles are assumed to be launched from the origin over a horizontal surface unless stated otherwise.

A projectile is fired at a speed of 840 m/sec at an angle of 60 degrees. How long will it take to get 21 km downrange?

Solution:

The projectile will get downrange, x units, in $t = \frac{x}{v_0 \cos\alpha}$, where α is the angle from the horizontal.

$$t = \frac{21,000}{(840)\cos\frac{\pi}{3}} = \frac{21,000}{(840)\frac{1}{2}} = 50s$$

This question was awarded 15 points using the correct formula and getting the correct final answer.

5. Section 13.3: 6

Find the curve's unit tangent vector. Also, find the length of the indicated portion of the curve.

$$r(t) = 6t^3\vec{i} - 2t^3\vec{j} - 3t^3\vec{k}, 1 \leq t \leq 2$$

Solution:

$$\vec{T} = \frac{r'(t)}{||r'(t)||} = \frac{18t^2\vec{i} - 6t^2\vec{j} - 9t^2\vec{k}}{\sqrt{(18t^2)^2 + (6t^2)^2 + (9t^2)^2}}$$

$$= \frac{18t^2\vec{i} - 6t^2\vec{j} - 9t^2\vec{k}}{\sqrt{(324+36+81)t^2}} = \frac{18t^2\vec{i} - 6t^2\vec{j} - 9t^2\vec{k}}{21t^2}$$

$$= \frac{6}{7}\vec{i} - \frac{2}{7}\vec{j} - \frac{3}{7}\vec{k}$$

Getting the correct vector was worth 10 points. Showing work for the magnitude of $r'(t)$ was a factor considered in awarding points for this part and the part below.

$$\int_1^2 ||r'(t)||\mathrm{d}t = \int_1^2 \sqrt{(18t^2)^2 + (6t^2)^2 + (9t^2)^2}\mathrm{d}t = \int_1^2 21t^2\mathrm{d}t = 7t^3\Big|_1^2$$

$$7(2)^3 - 7(1)^3 = 49$$

Correctly evaluating the integral was worth 5 points. If you did not show work calculating $||r'(t)||$ points were deducted.

6. Section 13.4: 11

Find T, N, and k for the space curve.

$$r(t) = (e^t \cos t)\vec{i} + (e^t \sin t)\vec{j} + 2\vec{k}$$

Solution:

$$\vec{T} = \frac{r'(t)}{||r'(t)||} = \frac{(e^t(\cos t - \sin t))\vec{i} + (e^t(\sin t + \cos t))\vec{j} + 0\vec{k}}{\sqrt{(e^t(\cos t - \sin t))^2 + (e^t(\sin t + \cos t))^2 + 0^2}}$$

$$= \frac{(e^t(\cos t - \sin t))\vec{i} + (e^t(\sin t + \cos t))\vec{j}}{\sqrt{e^{2t}(\cos^2 t - 2\sin t\cos t + \sin^2 t) + e^{2t}(\cos^2 t + 2\sin t\cos t + \sin^2 t)}}$$

$$= \frac{(e^t(\cos t - \sin t))\vec{i} + (e^t(\sin t + \cos t))\vec{j}}{\sqrt{2e^{2t}}}$$

$$= \frac{\sqrt{2}}{2}(\cos t - \sin t)\vec{i} + \frac{\sqrt{2}}{2}(\sin t + \cos t)\vec{j}$$

Correctly calculating the unit tangent was awarded 5 points.

$$\vec{N} = \frac{T'(t)}{||T'(t)||} = \frac{-\frac{\sqrt{2}}{2}(\cos t + \sin t)\vec{i} + \frac{\sqrt{2}}{2}(\cos t - \sin t)\vec{j}}{\sqrt{\frac{1}{2}\left[(\cos t + \sin t))^2 + (\cos t - \sin t))^2\right]}}$$

$$= \frac{-\frac{\sqrt{2}}{2}(\cos t + \sin t)\vec{i} + \frac{\sqrt{2}}{2}(\cos t - \sin t)\vec{j}}{1} = -\frac{\sqrt{2}}{2}(\cos t + \sin t)\vec{i} + \frac{\sqrt{2}}{2}(\cos t - \sin t)\vec{j}$$

Correctly calculating the unit normal was awarded 5 points. Calculating the magnitude of the $T'(t)$ was a part of the 5 points. Some explanations were sufficient.

$$\kappa = \frac{\left\|\vec{T}'(t)\right\|}{\|\vec{r}'(t)\|} = ... = \frac{1}{\sqrt{2}e^t} = \frac{\sqrt{2}e^{-t}}{2}$$

Showing how you obtain curvature and getting the correct answer was awarded 5 points.

Common Mistakes and feedback:

- Read the directions to make sure you are doing all the parts.
- Skipped steps. This was particularly important when calculating the curvature.
- Changing vector notation led to mistakes. Stick with one. Emphasize those pointy brackets.
- Show evaluation of limits, derivatives, and integrals. Show the calculus.
- Staple your work so you do not lose any papers.
- Always calculate the magnitude of $T'(t)$ when calculating $N'(t)$ because it is NOT always one.

- If you submitted your paper online, there was no physical feedback to give, only your grade.

If you believe there were any mistakes, or have questions about how I graded your work, feel free to email me or show up to the LA sessions.