14.3-14.7 HOMEWORK

Graded by Learning Assistant Enzo De Oliveira.

I numbered the problems from each section and used a random number generator to select one problem from each section for 5 problems total.

The questions graded were

- Section 14.3: 71
- Section 14.4: 27
- Section 14.5: 11
- Section 14.6: 5
- Section 14.7: 33

Each question was out of 16 points, for a total of 80 points for accuracy. Another 20 points was awarded for neatness, legibility, work, stapling etc.

1. Section 14.3: 71

Find a function $z = f(x, y)$ whose partial derivatives are as given.

$$\frac{\partial f}{\partial x} = 3x^2y^2 - 2x \text{ and } \frac{\partial f}{\partial y} = 2x^3 + 6y$$

Solution:

By choosing $\frac{\partial f}{\partial x}$ and integrating with respect to x we get,

$$f(x, y) = x^3y^2 - x^2 + c(y)$$

where $c(y)$ is some function of y.

Now differentiating with respect to y we obtain,

$$\frac{\partial f}{\partial y} = 2yx^3 + c'(y) = 2x^3y + 6y$$

So

$$c'(y) = 6y$$

and

$$c(y) = 3y^2 + c$$

where c is an arbitrary constant.

Therefore

$$f(x, y) = x^3y^2 - x^2 + 3y^2 + c$$

.

2. Section 14.4: 27

Assuming y is a differentiable function of x, use Theorem 8 to find the value of $\frac{dy}{dx}$ at the given point.

$$x^2 + xy + y^2 - 7 = 0, (1, 2)$$

Solution:
Theorem 8 states that

$$\frac{dy}{dx} = -\frac{F_x}{F_y}$$

So

$$F_x = 2x + y$$

$$F_y = x + 2y$$

$$\frac{dy}{dx} = -\frac{2x + y}{x + 2y}$$

$$\left.\frac{dy}{dx}\right|_{(x,y)=(1,2)} = -\frac{2(1) + (2)}{(1) + 2(2)} = -\frac{4}{5}$$

3. Section 14.5: 11

Find the derivative of the function at P_0 in the direction of $\mathbf{u}$.

$$f(x, y) = 2xy - 3y^2, P_0(5, 5), \mathbf{u} = 4\mathbf{i} + 3\mathbf{j}$$

Solution:

$$\mathbf{v} = \frac{\mathbf{u}}{|\mathbf{u}|} = \frac{4}{5}\mathbf{i} + \frac{3}{5}\mathbf{j}$$

$$\nabla f(x, y) = \langle 2y, 2x - 6y\rangle$$

$$\nabla f(5, 5) = \langle 10, -20\rangle$$

$$\nabla f(5, 5) \cdot \mathbf{v} = 10 \cdot \frac{4}{5} - 20 \cdot \frac{3}{5} = -4$$

4. Section 14.6: 5

Find the equations for the **(a)** tangent plane and **(b)** normal line at the point P_0.

$$\cos(\pi x) - x^2y + e^{xz} + yz = 4$$

Solution:

(a)

$$f_x = -\pi\sin(\pi x) - 2xy + ze^{xz}$$

$$f_x(P_0) = \pi\sin(\pi(0)) - 2(0)(1) + (2)e^{(0)(2)} = 0 - 0 + 2 = 2$$

$$f_y = -x^2 + z$$

$$f_y(P_0) = -(0)^2 + 2 = 2$$

$$f_z = xe^{xz} + y$$

$$f_z(P_0) = (0)e^{(0)(2)} + 1 = 1$$

Therefore the equation for the plane is

$$2(x-0) + 2(y-1) + (z-2) = 0$$

$$2x + 2y + z = 4$$

(b) Using the work from before the equation of the line is

$$x = 2t, y = 1 + 2t, x = 2 + t$$

5. Section 14.7: 33

Find the absolute maxima and minima of the functions on the given domain.

$f(x, y) = x^2+y^2$ on the closed triangular plate bounded by the lines $x = 0, y = 0, y+2x = 2$ in the first quadrant.

Solution:

First find the interior points.

$$f_x = 2x$$

$$f_x = 0 \rightarrow x = 0$$

$$f_y = 2y$$

$$f_y = 0 \to y = 0$$

So we obtain the point $(0, 0)$ and $f(0, 0) = 0$.

Next we find the boundary points.

Along $x = 0$

$$f(0, y) = y^2$$

The intersection between $y = 2 - 2x$ and $x = 0$ means y ranges from 0 to 2.

Which means $f(0, 2) = 2^2 = 4$ and $f(0, 0) = 0^2 = 0$

Along $y = 0$

$$f(x, 0) = x^2$$

The intersection between $y = 2 - 2x$ and $y = 0$ means x ranges from 0 to 1.

Which means $f(1, 0) = 1^2 = 1$ and $f(0, 0) = 0^2 = 0$.

Along $y = 2 - 2x$

$$f(x, 2 - 2x) = x^2 + (2 - 2x)^2$$

$$f(x) = 5x^2 - 8x + 4$$

$$f'(x) = 10x - 8$$

$$f'(x) = 0 \to x = \frac{4}{5}$$

$$y = 2 - 2(\frac{4}{5}) \to y = \frac{2}{5}$$

$$f(\frac{4}{5}, \frac{2}{5}) = \frac{4}{5}$$

Therefore the absolute maximum is 4 at (0,2) and the absolute minima is 0 at (0,0).

Common Mistakes and feedback:

- Read the directions to make sure you are doing all the parts.
- Show evaluation! Prove you know whaat you are doing.
- Changing vector notation led to mistakes. Stick with one. Emphasize those pointy brackets.
- Explaain your work when needed to get rid of ambiguity.
- Staple your work so you do not lose any papers.
- For finding extrema, make sure to plug in the boundary equation so you can solve 1 variable optimization.
- If you submitted your paper online, there was no physical feedback to give, only your grade.

If you believe there were any mistakes, or have questions about how I graded your work, feel free to email me or show up to the LA sessions.