14.8-15.4 HOMEWORK

Graded by Learning Assistant Enzo De Oliveira.

I numbered the problems from each section and used a random number generator to select one problem from each section for 5 problems total.

The questions graded were

- Section 14.8: 3
- Section 15.1: 21
- Section 15.2: 31
- Section 15.3: 1
- Section 15.4: 11

Each question was out of 16 points, for a total of 80 points for accuracy. Another 20 points was awarded for neatness, legibility, work, stapling etc.

1. Section 14.8: 3

Find the maximum value of $f(x, y) = 49 - x^2 - y^2$ on the line $x + 3y = 10$.

Solution:

First, establish that $g(x, y) = x + 3y - 10 = 0$. Second, the two conditions that need to be met are $\nabla f = \lambda \nabla g$ and $g(x, y) = 0$.

$$\nabla f = \lambda \nabla g$$

$$-2x\mathbf{i} - 2y\mathbf{j} = \lambda\mathbf{i} + 3\lambda\mathbf{j}$$

$$\lambda = -2x \text{ and } 3\lambda = -2y$$

$$x = -\frac{\lambda}{2} \text{ and } y = -\frac{3\lambda}{2}$$

Plugging into $g(x, y) = 0$

$$\left(-\frac{\lambda}{2}\right) + 3\left(-\frac{3\lambda}{2}\right) = 0$$

$$\lambda = -2$$

Plugging back into our λ equations for x and y

$$x = 1 \text{ and } y = 3$$

Absolute maximum:

$$f(1, 3) = 49 - 1^2 - 3^2 = 39$$

2. Section 15.1: 21

Evaluate the double integral over the given region:

$$\iint_R e^{x-y}\, dA$$

R: $0 \le x \le \ln 2, 0 \le y \le \ln 2$

Solution:

$$\int_0^{\ln 2}\int_0^{\ln 2} e^x \cdot e^{-y}\, dx\, dy$$

$$\int_0^{\ln 2} e^x\, dx \int_0^{\ln 2} e^{-y} \cdot dy$$

$$\left(e^x\Big|_0^{\ln 2}\right) \cdot \left(-e^{-y}\Big|_0^{\ln 2}\right)$$

$$(2-1) \cdot (-\frac{1}{2} + 1) = \frac{1}{2}$$

3. Section 15.2: 31

Given is an integral over a region in a Cartesian coordinate plane. Sketch the region and evaluate the integral.

$$\int_{-\pi/3}^{\pi/3}\int_0^{\sec t} 3\cos t\, du\, dt$$

Solution:

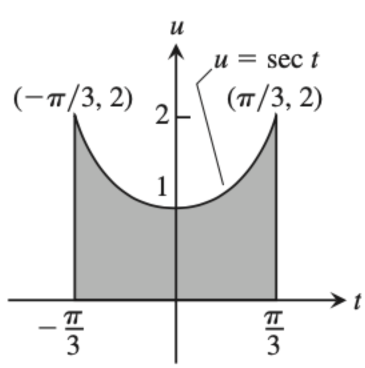

Figure 1. **Region**

$$\int_{-\pi/3}^{\pi/3}\int_0^{\sec t} 3\cos t\,du\,dt$$

$$3\int_{-\pi/3}^{\pi/3} \cos t\Big|_0^{\sec t}\,dt$$

$$3\int_{-\pi/3}^{\pi/3} \cos t\cdot\sec t\,dt$$

$$3\Big[t\Big]_{-\pi/3}^{\pi/3}$$

$$3(\frac{\pi}{3}+\frac{\pi}{3}) = 2\pi$$

4. Section 15.3: 1

Sketch the region bounded by the given lines and curves. Then express the region's area as an iterated double integral and evaluate the integral.

The coordinate axes and the line $x + y = 2$

Solution:

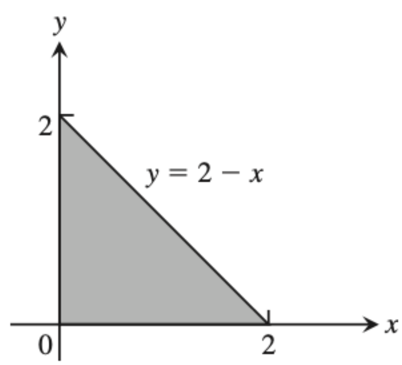

Figure 2. **Region**

$$\int_0^2\int_0^{2-y} 1\,dx\,dy$$

$$\int_0^2 2 - y\,dy$$

$$2y - \frac{1}{2}y^2\Big|_0^2$$

$$4 - \frac{1}{2}(4) = 4 - 2 = 2$$

5. Section 15.4: 11

Change the Cartesian integral into an equivalent polar integral. Then evaluate the polar integral.

Solution:

$$\int_0^2 \int_0^{\sqrt{4-y^2}} (x^2 + y^2)\, dx\, dy$$

$$\int_0^2 \int_0^{\sqrt{4-y^2}} x^2 + y^2\, dx\, dy$$

$$\int_0^{\pi/2} \int_0^2 r^2 \cdot r\, dr\, d\theta$$

$$\int_0^{\pi/2} 1\, d\theta \int_0^2 r^2 \cdot r\, dr$$

$$\left(\frac{\pi}{2}\right) \cdot \left(\frac{1}{4} r^4 \Big|_0^2\right)$$

$$\left(\frac{\pi}{2}\right) \cdot (4) = 2\pi$$

Common Mistakes and feedback:

- Read the directions to make sure you are doing all the parts.
- Show evaluation! Prove you know what you are doing.
- When evaluating iterated integral do not do two separate integrals, evaluate one inside the others.
- Explain your work when needed to get rid of ambiguity.
- Staple your work so you do not lose any papers.
- Label your graphs to correctly define the function's sketch.
- If you submitted your paper online, there was no physical feedback to give, only your grade.

If you believe there were any mistakes, or have questions about how I graded your work, feel free to email me or show up to the LA sessions.