MAC 2313 Feb 23, 2017
Exam II Prof. S. Hudson

1) [5 pts] Find the intersection of the line defined below with the xz-plane.
$x = -2$
$y = 4 + 2t$
$z = -3 + t$

2) [5 pts] Compute the limit as $(x, y) \to (0, 0)$, $\lim \frac{xy}{\sqrt{x^2+y^2}}$, showing all work and reasoning.

3) [10 pts] Convert from spherical to rectangular coordinates: $\rho = 13 \sec \phi$.

4) [7 pts] Evaluate the definite integral $\int_1^2 (t^3\mathbf{i} + t^2\mathbf{j})dt$

5) [10 pts] Find the radius of curvature of this curve at $t = \pi/2$. Simplify completely.

$x = 3\cos t$, $y = 4\sin t$, $z = t$

6) with minor typo's corrected 2/25/17 [10 pts, mostly for 6b] 6a) Write out the usual formula for acceleration including a_T and a_N, with a simple 2D sketch of what the vectors in it might look like.

6b) Find the normal scalar component of acceleration when $t = 1$, given that
$\mathbf{a}(1) = \mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ and $a_T(1) = 2$.

7) [10 pts] Use a total differential (df) to approximate the change in the value of $f(x, y) = x^{1/3}y^{1/2}$ from $P(8, 9)$ to $Q(7.78, 9.03)$. Simplify completely.

8) [8 pts] Sketch the level curve $z = -4$ for $z = f(x, y) = x^2 - y^2$. As usual, the 'curve' may have more than one component.

9) [15 pts] Answer T or F; you do not have to justify your answers (but this sometimes helps, if there is some minor misunderstanding):

If every point in D is an interior point, then D is an open set.

If some point in D is not an interior point, then D is a closed set.

For a moving particle, the unit tangent vector and the acceleration vector are parallel.

If f_x and f_y exist and are continuous, then $f(x, y)$ is differentiable.

If f_{xy} and f_{yx} exist and are continuous on an open set, then $f_{xy} = f_{yx}$ there.

10) [10 pts] State the definition of *differentiable* for a function $f(x, y)$ at a point (x_0, y_0). Hint: this involves a limit and seven $\triangle$'s, more or less. As always, include enough words.

11) [10 pts] Choose ONE proof:

A) Theorem 12.6.2, that $\mathbf{a} = s''(t)\mathbf{T} + \kappa(s'(t))^2\mathbf{N}$.

B) Theorem 12.6.3b, that $a_N = \frac{||\mathbf{a}\times\mathbf{v}||}{||\mathbf{v}||}$.

C) Assuming $\mathbf{T}'(t) \neq \mathbf{0}$, show $\mathbf{N}(t) \perp \mathbf{T}(t)$. Use the definition of $\mathbf{N}(t)$.

Bonus [about 5 points]: Use the definition to show that $f(x, y, z) = x^2 + y^2 + z^2$ is differentiable at $P(1, 2, 3)$. This is a bit harder than the HW examples.

Remarks and Answers: The average score was 68% with highs of 87 and 87. The best results were on problem 4 (87%) and the worst were on problems 6 and 8 (45% and 33%). The advisory scale for the exam is

A's 76 to 100
B's 66 to 75
C's 56 to 65
D's 46 to 55

To estimate your semester grade so far, average your scores on exams 1 and 2, and use that number with the scale below. You may also factor in HW if you like, but it is hard to do that precisely at this point.

A's 78 to 100
B's 68 to 77
C's 58 to 67
D's 48 to 57

1) $(-2, 0, -5)$. Set $y = 0$, get $t = -2$ and then $z = -5$.

2) 0. But the grade also depended on your reasoning. If you computed the limit from several directions (for example, set $y = x$, etc), I gave full credit. Using polar coordinates is slightly better (but even this method does not completely *prove* that the limit exists!)

This is a 0/0 type limit, but it is not a single-variable limit as in Calc I. There is no L'Hopital's Rule for this (as far as I know). But it is possible that your calculations could lead to a single-variable limit, so that L'Hopital's Rule could be used later in the problem.

3) $z = 13$. Hopefully you remember that z, ϕ and ρ are closely related, $z = \rho\cos\phi = 13\sec\phi\cos\phi = 13$. If you have trouble remembering the new identities, draw the standard right triangle with sides z, r and ρ and use trig.

4) $15/4\ \mathbf{i} + 7/3\ \mathbf{j}$

5) $r = 5/2$. Use $\kappa = \frac{||\mathbf{a}\times\mathbf{v}||}{||\mathbf{v}||^3} = \cdots = \sqrt{160}/\sqrt{1000} = 2/5$. Common mistakes were a) not using $t = \pi/2$ and b) stopping at 2/5.

6a) $\mathbf{a} = a_T\mathbf{T} + a_N\mathbf{N}$ (though I accepted other similar formulas). The sketch should include a right triangle, which might remind you of the Pythagoras formula, which becomes $||\mathbf{a}||^2 = a_T^2 + a_N^2$. This could help you with part b).

6b) $\sqrt{5}$ from the Pythagoras formula and the given info. Note that this problem is a little unusual in that you are not given a formula for $\mathbf{r}(t)$, so you cannot compute $\mathbf{v} = \mathbf{r}'(t)$, etc.

7) $df = f_x\Delta x + f_y\Delta y = -0.22/4 + 0.03/3 = -0.45$.

8) Graph $-4 = x^2 - y^2$ in the xy-plane. It is a hyperbola (2 separate parts) through $(0, 2)$ and $(0, -2)$. The top part should be concave up.

9) TFFTT

10) See the text. I accepted the equation by itself.

11) See the text. The proofs on this exam were mostly calculations, but even so, you should explain any key steps:

A) Include $v = (ds/dt)T$ near the start. Mention the product rule and chain rule.

B) Include the picture, and (unless your picture is exceptionally clear) explain how $\sin\theta$ comes in.

C) Mention (but do not prove) the theorem about $r \cdot r' = 0$.

Bonus) We need to compute a limit of $\frac{\Delta f - df}{||PQ||}$. A few people got partial credit but most got stuck pretty early, often on Δf. Briefly, $df = 2(x-1) + 4(y-2) + 6(z-3)$ and $\Delta f = x^2 + y^2 + z^2 - (1^2 + 2^2 + 3^2)$. Now algebra (such as $y^2 - 4 - 4(y-2) = (y-2)^2$) leads to $\Delta f - df = (x-1)^2 + (y-2)^2 + (z-3)^2 = ||PQ||^2$. So, the definition leads to lim $\frac{||PQ||^2}{||PQ||} = 0$ as desired.