MAC 2313 Feb 27, 2019
Exam II Prof S. Hudson

1) [8pts] Let $z = \sin(y^3 - 5x)$. Find the rate of change of z with respect to x at the point $(0, 1)$ with y held fixed.

2) [8pts] Let $f(x, y) = e^{xy^2}$. Find $f_{xy}(1, 2)$.

3) [8pts] Let $f(x, y) = 3x^2y^2$ and $\mathbf{u} = \mathbf{j}$. Compute the directional derivative $D_{\mathbf{u}}f(1, 1)$.

4) [8pts] Given that $\mathbf{a}(t) = 2t\mathbf{i} + \cos(t)\mathbf{j} + e^t\mathbf{k}$ and $\mathbf{v}(0) = \mathbf{j}$, find the velocity vector for the particle as a function of t.

5) [8pts] You are given that $\mathbf{v} = -4\mathbf{j}$ and $\mathbf{a} = 2\mathbf{i} + 7\mathbf{j}$ at a certain instant of time. Find a_T and T at that time. For a little extra credit, find a_N and N.

6) [10 pts] Find the arc length parametrization of the circular helix

$$\mathbf{r} = \cos t\mathbf{i} + \sin t\mathbf{j} + 4t\mathbf{k}$$

that has reference point $\mathbf{r}(0) = (1, 0, 0)$ and has the same orientation as the given helix.

7) [10 pts] Find the curvature and radius of curvature for this curve at $t = \pi/2$. Simplify completely.

$x = 3\sin t,\ y = 4\cos t,\ z = 3t.$

8) [10 pts] Let $f(x, y) = 2xy^2$. Use a total differential to approximate the change in f from $P(1, 4)$ to $Q(0.99, 4.02)$. Simplify completely.

9) [20 pts] Answer T or F; you do not have to justify your answers.

The function defined by $f(x, y) = \frac{xy}{x^2+y^2}$ and $f(0, 0) = 0$ is differentiable at $(0, 0)$.

The natural domain of $f(x, y) = \ln(y - x)$ is an open subset of R^2.

The natural domain of $f(x, y, z) = \sqrt{1 - x^2 - y^2 - z^2}$ is an bounded subset of R^3.

If $f(x, y) = x^2 + y^2$ then $f_{xx}(x, y) > 0$ for all (x, y).

If f_{xy} and f_{yx} both exist at (x, y) then $f_{xy} = f_{yx}$ at (x, y).

If $\mathbf{r}(t)$ parametrizes a curve C and it is differentiable then it is a smooth parametrization.

If D is an closed set in R^2 then every point in D is a boundary point.

Gradients are tangent to level curves.

By definition, the binormal vector is $\mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t)$.

The curvature of C given by a smooth arclength parametrization $\mathbf{r}(s)$ is $\kappa = ||d\mathbf{T}/ds||$.

10) [10 pts] Choose ONE proof:

A) Theorem 12.6.2, that $\mathbf{a} = s''(t)\mathbf{T} + \kappa(s'(t))^2\mathbf{N}$.

B) State and prove Theorem 12.6.3b, the formula for a_N using $\mathbf{v}$ and $\mathbf{a}$. Include a picture and enough words.

Remarks and Answers: The average grade among the top 20 students was approx 66 out of 100, which is normal. The two highest scores were 94 and 88. The results were low on problem 10, the proof, approx 40%. The results were slightly above 80% on problems 3, 5 and 8. Here is an advisory scale for Exam II :

A's 73 to 100
B's 63 to 72
C's 53 to 62
D's 43 to 52

To estimate your semester grade quickly, average your two exam scores and use a scale halfway between the Exam I scale and the Exam II scale (so A's start at 72, etc). It is hard to include the HW scores precisely yet, but you might adjust your average a little for that, maybe ± 5 points at most. If your current grade is below a D (an exam average under 47 or so) it is very unlikely you will raise it to a C. You should consider dropping the course by next week and trying again.

There were three versions of this exam, which differed slightly only in problems 5, 6 and 8. The main answers below are meant for exams that match the questions above. But I included secondary answers below for the alternate versions.

1) $z_x = -5\cos(1)$.

2) $f_{xy}(1,2) = 20e^4$

3) $D_u f = \nabla f \cdot \mathbf{j} = \langle 6,6\rangle \cdot \langle 0,1\rangle = 6$.

4) Use the FTC to get $\mathbf{v}(t) - \mathbf{v}(0) = \int_0^t \mathbf{a} = \langle t^2,\ \sin(t),\ e^t\rangle - \langle 0,0,1\rangle$. Solve for $\mathbf{v}(t)$ and simplify to get $\mathbf{v}(t) = \langle t^2,\ \sin(t)+1,\ e^t - 1\rangle$.

5) $\mathbf{T} = -\mathbf{j}$ and $a_T = \frac{\mathbf{v}\cdot\mathbf{a}}{||\mathbf{v}||} = -7$. For a small bonus, $a_N = 2$ by using a cross product, and $\mathbf{N} = \mathbf{i}$ by solving for $\mathbf{N}$ in $\mathbf{a} = a_T\mathbf{T} + a_N\mathbf{N}$. This answer is based on $\mathbf{a} = 2\mathbf{i} + 7\mathbf{j}$, which appeared on one-third of the exams. If your exam has $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j}$ instead, then $a_T = -3$ (with no change to the rest). If your exam has $\mathbf{a} = 2\mathbf{i} + 5\mathbf{j}$, then $a_T = -5$.

6) If your exam has a formula for $\mathbf{r}$ that includes $+4t\mathbf{k}$, use $s(t) = \int_0^t ||\mathbf{r}'(T)||\ dT = \int_0^t \sqrt{17} = \sqrt{17}t$ to get $t = s/\sqrt{17}$. Then $\mathbf{r}(s) = \cos(s/\sqrt{17})\mathbf{i} + \sin(s/\sqrt{17})\mathbf{j} + (4s/\sqrt{17})\mathbf{k}$. If your exam includes $+3t\mathbf{k}$ instead, replace $s = \sqrt{17}t$ with $s = \sqrt{10}t$ (etc). With $+2t\mathbf{k}$, use $\sqrt{5}$.

7) $\kappa = \frac{||\mathbf{a}\times\mathbf{v}||}{||\mathbf{v}||^3} = 15/5^3 = 3/25$, so radius $= 25/3$. For simplicity, you should plug in $t = \pi/2$ asap after taking derivatives. For example, $\mathbf{a} = \langle -3\sin(t), -4\cos(t), 0\rangle = \langle -3, 0, 0\rangle$.

8) Use $df = f_x dx + f_y dy = 2y^2(-0.01) + 4xy(0.02)$. If your exam has $P(1,4)$ get an answer of $df = 0$. With $P(1,3)$ get $df = 0.06$. With $P(1,5)$ get $df = -.10$.

9) FTTTF FFFTT

10) See the textbook for the proofs. Part B) is probably easier and most people chose it and did OK, at least with the equations. The most common problems were failure to study these and lack of explanation. For example, it is important in the proof that θ is the angle between **a** and **v** (or **T**). So write that out. Your picture should also show this clearly, with all lines clearly labeled.