MAC 2313 Oct 22, 2020
Exam II Key Prof. S. Hudson

Problems 1-6 are 7pts each. Problems 7-11 are 8pts each.

1) Let $f(x, y) = xe^y + y + 1$. Find all four second order partial derivatives of f.

2) The speedometer on your car reads a steady 35 mph. Could you be accelerating ? Explain.

3) Determine whether the limit exists. If so, find its value.

$$\lim_{(x,y)\to(0,0)} \frac{3xy}{5x^2 + 4y^2}$$

4) Let $w = xy + yz + xz$, $x = u + v$, $y = u - v$ and $z = uv$. Use the Ch.14.4 Chain Rule to find $\frac{\partial w}{\partial u}$ at the point $(1/2, 1)$.

5) Find the local minima, the local maxima and the saddle points of $f(x, y) = e^{-y}(x^2 + y^2)$.

6) Use Lagrange multipliers to find the extreme values of $f(x, y) = x - 3y - 1$, subject to the constraint $x^2 + 3y^2 = 16$.

7) Find the equation of the tangent plane to the surface $2z - x^2 = 0$ at the point (2,0,2).

In the next 3 related problems, let $f(x, y) = \ln(1 + 2x + 3y)$.

8) Compute $\frac{\partial f}{\partial x}(0, 0)$ and $\frac{\partial f}{\partial y}(0, 0)$. Suggestion: do this carefully, or check your answer.

9) Given that $f(0, 0) = \ln 1 = 0$, use a differential to estimate $f(.02, -0.01)$. [This kind of problem is sometimes phrased as "Use a local linear approximation . . ." or "Use linearization . . ." but they mean roughly the same thing and you can follow whichever wording makes sense to you.]

10) Find a unit vector for the direction in which f increases most rapidly at $(0, 0)$ and the rate of increase in this direction.

11) Find the absolute maximum and the absolute minimum values of $f(x, y) = x^2 + xy + y^2 - 6x + 2$ on the rectangular plate $0 \le x \le 5$, $-3 \le y \le 0$. Hints to save you some work - the extrema do *not* occur on the line $x = 5$, nor on the line $y = 0$. You should study the rest of the domain carefully, as usual, showing all work.

12) [18 points] True - False: Assume f, f_x etc are differentiable on R^n.

$\int_1^5 \int_3^4 f(x, y)dxdy = \int_1^4 \int_3^5 f(x, y)dydx$

If $f(x, y) = 7$ at all points on the curve $y = x^2$ then $f_y(0, 0) = 0$.

If P is a critical point of f and $D = f_{xx}f_{yy} - (f_{xy})^2 > 0$ at P, then f has a rel. min. at P.

Suppose $\mathbf{u} = \frac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j})$ and that $D_{\mathbf{i}}f(P) = 0$ and $D_{\mathbf{j}}f(P) > 0$. Then $D_{\mathbf{u}}f(P) > 0$.

Always $a_N \geq 0$ (the notation here is from Ch.13.5, the motion section).

If $f(x, y)$ is continuous on a closed rectangle R, then f has a maximum value on R.

Bonus) [approx 5 pts] State the main formula (it includes a scalar product) for the directional derivative $D_u f(P)$ and prove it.

Remarks, Scale, Answers: The average was approx 72% with high scores of 92 and 90, which is pretty good. The best scores were on problems 1, 7, 8 and 10 (over 85% on each) and the worst were on 3 and 5 (35% on each). Your exam score is written in orange at the top-center of page 1. Here is an advisory scale for Exam 2:

A's 80 - 100
B's 70 - 79
C's 60 - 69
D's 50 - 59

Combining all the quiz and exam grades, but not yet the HW, I wrote your semester average in black in the upper right just above the date. The 4 quizzes are 2 points each, Exam 1 is 15 and Exam 2 is 20 (total points = 43). You should check my number, which will also check that we agree on your scores. The class average of these numbers is 65% with highs of 89 and 87. Here is an advisory scale for the semester so far:

A's 73 - 100
B's 63 - 72
C's 53 - 62
D's 43 - 52

I think the drop date is Nov 2 (but check). If you need advice, see me or email me. Roughly, I'd say you have a good chance to pass with a C if your current average is at least 50, and you have decent HW grades, and you plan to work hard. If your current average is below 43 you might pass, but the odds seem low. Answers -

1) $f_{xx} = 0$, $f_{xy} = e^y$, $f_{yx} = e^y$, $f_{yy} = xe^y$.

2) Yes. The acceleration vector will have a non-zero normal component if the road is curved. I accepted most explanations that mentioned a curve, or the direction of velocity, for example.

3) DNE. You can explain this by finding 2 paths with different limits. For example, when $y = 0$ and $x > 0$ the limit is 0. When $y = x$ and $x > 0$ the limit is 1/3. Some answers lost points because the notation was wrong or the paths were not stated.

4) The chain rule leads to $w_u = (y + z) + (x + z) + (x + y)v$. Plugging-in, we get $w_u = 3$.

5) Set $\nabla f = \mathbf{0}$ and get the points $(0, 0)$ and $(0, 2)$. Using D deduce that (0,0) is a local min and that (0,2) is a saddle point. In hindsight, it is obvious that (0,0) gives the absolute min (since $f \geq 0$).

6) $\nabla f = \lambda \nabla g$ gives $1 = 2\lambda x$ and $-3 = 6\lambda y$. Conclude that $y = -x$. It is probably best not to solve for λ. Using the constraint, we get $x = \pm 2$, so the stationary points are (2,-2) and (-2,2). The extreme values are -9 and 7.

The E.V.Thm promises two values, so if you only find one, you should check for errors.

7) Set $\mathbf{n} = \nabla F = \langle -2x, 0, 2\rangle = \langle -4, 0, 2\rangle$. The equation is $-4(x-2) + 2(z-2) = 0$. Don't forget the "=0" part in your answer, or it is not an equation.

8) 2 and 3.

9) The main formula is $f(x, y) \approx f(x_0, y_0) + f_x dx + f_y dy$ which gives $0+2(0.02)+3(-0.01) = 0.01$. Here $dx = x - x_0 = 0.02 - 0.00 = 0.02$, etc.

10) $\mathbf{u} = \frac{1}{\sqrt{13}}\langle 2, 3\rangle$, by normalizing ∇f.

A few people went on to compute the maximal derivative ($\sqrt{13}$), and sometimes they circled it as the final answer. I could not give full credit for that, or for ambiguous versions of that. To be clear, stop when you have found the answer, and if it is not at the bottom for some reason, circle it.

11) From $\nabla f = \mathbf{0}$, get (4,-2). From $y = -3$, get (0,-3) and (4.5,-3). From the hint, ignore (0,0), etc.

Answers: max of 11 at (0,-3). min of -10 at (4,-2). Note that the question is about the *values*, not just where they occur, so include the 11 and -10 in your answer.

12) FFFTTT

Bonus) I mainly wanted to reward people who studied the proof list. I did not give partial credit for just the statement. See the text for the proof.