MAC 2313 — Nov 13, 2017
Exam III + Key — Prof. S. Hudson

1) Find the directional derivative of $f(x,y) = y^2 \ln x$ at $P(1,4)$ in the direction of $\mathbf{a} = -3\mathbf{i} + 3\mathbf{j}$.

2) Find parametric equations for the normal line to the surface $x^2y - 4z^2 = -7$ at $P(-3,1,-2)$.

3) Find parametric equations for the surface $z = e^{-x^2-y^2}$ in terms of r and θ (as defined in cylindrical coordinates).

4) Let G be the solid inside the sphere $x^2+y^2+z^2 = 9$ but outside the cylinder $x^2+y^2 = 1$. Express the volume of G as a double integral in polar coordinates. You do not have to evaluate it.

5) [15pts] Evaluate this integral by reversing the order of integration. Include a picture of the region R.

$$\int_0^2 \int_{y/2}^1 e^{x^2}\ dx\ dy$$

6) [15 pts] At what point(s) on the circle $x^2 + y^2 = 1$ does the function $f(x,y) = xy$ have an absolute maximum value and what is that max ? Hint: I'd suggest the Lagrange method, but a parametrization should also work.

7) Let $w = 5x^2y^3z^4$, $x = t^2$, $y = t^3$ and $z = t^5$. Compute dw/dt using a relevant version of the Chain Rule from Ch. 13.5. Write out the formula, use it, and for maximum credit simplify your answer.

8) [20pts] True-False. Assume f, f_x etc are differentiable on R^n (usually this is R^2).

The surface area of the paraboloid $z = x^2 + y^2$ below $z = 1$ is $\int_0^{2\pi} \int_0^1 \sqrt{4r^2+1}\ r\ dr\ d\theta$.

The function $f(x,y) = y^3$ has an absolute maximum value on the region $x^2 + y^2 < 4$.

If $y = x^2$ is a level curve of $f(x,y)$ then $f_x(0,0) = 0$.

If $f(x,y)$ is continuous on a closed rectangle R, then f has a maximum value on R.

In a Lagrange multiplier problem, the main goal is to compute λ as soon as possible.

If $\nabla f = \langle 3,\ 4\rangle$ at P and $\mathbf{u}$ is any unit vector, then $D_{\mathbf{u}} f(P) \le 5$.

$\int_0^1 \int_0^{2-2z} \int_0^2 x + y + z\ dx\ dy\ dz = \int_0^2 \int_0^2 \int_0^{1-y/2} x + y + z\ dz\ dx\ dy$.

$\int_0^1 \int_0^{2-2z} \int_0^2 x + y + z\ dx\ dy\ dz = \int_0^2 \int_0^1 \int_0^{1-z} x + y + z\ dy\ dz\ dx$.

If $D = f_{xx}f_{yy} - (f_{xy})^2 < 0$ at a critical point P, then f has a relative extremum at P.

Fubini's Theorem is mainly used to solve optimization problems.

Bonus) [about 5 points] State Thm.13.7.2 about the equation of the tangent plane to $z = f(x, y)$ at a point $P(x_0, y_0)$, and prove it using a level surface (as done in the book).

Remarks and Answers: The average was 86 out of 100, with top scores of 104 and 103, which is excellent. The overall results were good (at least 75%) on every problem. Here is an advisory scale for Exam 3:

A's 90 to 100
B's 80 to 89
C's 70 to 79
D's 60 to 69

Here is a scale for the average of your 3 exam scores so far.

A's 76 to 100
B's 66 to 75
C's 56 to 65
D's 46 to 55

1) $\nabla f \cdot \mathbf{u} = \langle 16, 0\rangle \cdot \langle -1/\sqrt{2}, 1/\sqrt{2}\rangle = -16/\sqrt{2}$, or $-8\sqrt{2}$. Don't forget to normalize **a** and that $\ln(1) = 0$.

2) $x = -3 - 6t$, $y = 1 + 9t$, $z = -2 + 16t$. This uses $\mathbf{n} = \nabla f(-3, 1, -2) = \langle -6, 9, 16\rangle$.

3) $x = r\cos\theta$, $y = r\sin\theta$ and $z = e^{-r^2}$.

4) $2\int_0^{2\pi}\int_1^3 \sqrt{9 - r^2}\; r\; dr\; d\theta$. Without the initial 2, we only get the top half. It is also OK to use more symmetry and get (for example) $8\int_0^{\pi/2}\int_1^3 \sqrt{9 - r^2}\; r\; dr\; d\theta$.

5) $e - 1$. We did this in class and I think it is also in the text.

6) Max $= 1/2$ at both $(x, y) = (\sqrt{1/2}, \sqrt{1/2})$ and $(-\sqrt{1/2}, -\sqrt{1/2})$. Most people used Lagrange (with $y = 2\lambda x$, $x = 2\lambda y$, etc). An alternative is $f = r^2 \sin\theta\cos\theta = (1/2)\sin 2\theta$, with $f' = 0$ at $\theta = 0, \pi/2, \ldots$.

7) $dw/dt = w_x\; dx/dt + w_y\; dy/dt + w_z\; dz/dt = \cdots = 165t^{32}$.

8) TFTTF TTFFF

B) See the text. Mention (in words!) that the gradient is orthogonal to the tangent plane and serves as the normal vector **n** of Ch.11.