MAC 2313 March 30, 2017
Exam III Prof. S. Hudson

1) Let $f(x,y) = 9 - x^2 - 7y^3$. Find the partial derivatives, $f_x(3,1)$ and $f_y(3,1)$.

2) Let $w = x^3y^2z$. Find the differential dw.

3) Find $\int\int_R \frac{1}{1+x^2+y^2}\, dA$ where R is the sector (region) in the first quadrant bounded by $y = 0$, $y = x$ and $x^2 + y^2 \le 4$.

4) Find parametric equations for the surface $z = e^{-x^2-y^2}$ in terms of r and θ (two cylindrical coordinates).

5) The surfaces $z = \sqrt{x^2 + y^2}$ (a cone) and the $x + 2y + 2z = 20$ (a plane) intersect in a curve C passing through the point $P(4, 3, 5)$.

a) Find the equation of the tangent plane to the cone at P.

b) Find parametric equations for the tangent line to the curve C at P.

6) Compute $\int\int_R 2x - y^2\, dA$, where R is the triangle bounded by $y = 3$, $y = -x + 1$ and $y = x + 1$.

7) [20 points] True - False: Assume f, f_x etc are differentiable on R^n.

$\int_1^2 \int_3^4 f(x,y)dxdy = \int_3^4 \int_1^2 f(x,y)dydx$

If $y = x^2$ is a contour (a level curve) of $f(x,y)$ then $f_x(0,0) = 0$.

If P is a critical point of f and $D = f_{xx}f_{yy} - (f_{xy})^2 > 0$ at P, then f has a rel. min. at P.

If G lies in the first octant, then the avg. value of $f(x,y) = x^2 - y + z$ on G is nonnegative.

If $f(x,y)$ is continuous on a closed region R, then f has a maximum value on R.

8) Use Lagrange multipliers to find the extreme values of $f(x,y) = x - 3y - 1$, subject to the constraint $x^2 + 3y^2 = 16$.

9) Prove (or explain carefully) ONE of these:

a) Thm 13.6.6 (that ∇f is normal to level curves of f).

b) Thm 13.9.3 (Lagrange's formula with two variables, that $\nabla f = \lambda \nabla g$)

Remarks and Answers: The average among the top 20 was approx 68, which is normal. The two best scores were 91 and 84. The average scores were similar for each problem except for problems 5 and 9 (approx 45% on each). The advisory scale is:

A's 75 to 100
B's 65 to 74
C's 55 to 64
D's 45 to 54

To estimate your current semester standing, average your three exam grades and use the scale below:

A's 71 to 100
B's 61 to 70
C's 51 to 60
D's 41 to 50

1) -6 and -21

2) $dw = 3x^2y^2z\ dx + 2x^3yz\ dy + x^3y^2\ dz$

3) $\frac{\pi \ln(5)}{8}$. Draw a picture, convert to polar coordinates, set $u = 1 + r^2$, etc.

4) $x(r, \theta) = r\cos\theta$, $y(r, \theta) = r\sin\theta$, $z(r, \theta) = e^{-r^2}$. Some people got equations like these, but mixed them in with irrelevant or incorrect formulas. I generally gave full credit if these 3 were grouped together at the bottom of the answer space and/or were circled, to clarify that they formed the final answer.

5a) Ignore the plane until 5b) and study only the cone. With $f(x, y) = \sqrt{x^2 + y^2}$ and $\mathbf{N} = \langle f_x, f_y, -1\rangle = \langle 4/5, 3/5, -1\rangle$, we get the equation

$$4(x - 4)/5 + 3(y - 3)/5 - (z - 5) = 0.$$

Simplification is not required. It is not OK to omit the '= 0' part. It is OK to use $F(x, y, z) = x^2 + y^2 - z^2 = 0$ instead, and get $\mathbf{N} = \nabla F = \langle 8, 6, -10\rangle$ which leads to an equivalent equation for the plane.

5b) A tangent vector to the curve will lie in both tangent planes, and must be orthogonal to both normal vectors. So let

$$\mathbf{T} = \mathbf{N} \times \mathbf{N_2} = \langle 8, 6, -10\rangle \times \langle 1, 2, 2\rangle = \langle 32, -26, 10\rangle$$

and get $x(t) = 4 + 32t$, $y(t) = 3 - 26t$, $z(t) = 5 + 10t$.

6) Based on a careful picture and a little easy algebra, get $\int_1^3 \int_{1-y}^{y-1} 2x - y^2\ dx\ dy = \cdots = 2\int_1^3 y^2 - y^3\ dy = \cdots = -68/3$.

R is an isosceles triangle, symmetric wrt to the y-axis, but $2x - y^2$ does not have that symmetry. It is possible to use symmetry, for a fancy shortcut, by splitting the integrand into an odd part and an even part. If interested in this option, study these:

$\int_1^3 \int_{1-y}^{y-1} 2x\ dx\ dy = 0$ and

$\int_1^3 \int_{1-y}^{y-1} y^2\ dx\ dy = 2\int_1^3 \int_0^{y-1} y^2\ dx\ dy = -68/3$.

7) TTFFF The 4th one is false because, for example, $f(1, 10, 1) < 0$ and G might contain only points like $(1, 10, 1)$.

8) Max is 7 at $(2, -2)$. Min is -9 at $(-2, 2)$. Some key steps are $1 = 2\lambda x$, $y = -x$, $x = \pm 2$, etc. Some people included $(2, 2)$ in the work. There is no reason to do so, since $y \neq -x$ there. But in the grading, I considered this a harmless mistake.

9a) It is best to stick to 2 dimensions ("curves") and use parametric equations, leading to $\mathbf{T} \cdot \nabla \mathbf{f} = 0$. The grades were either high or low. It seemed that either you studied this or you didn't, with not much middle ground.

9b) You were supposed to *justify* the formula, probably without the same rigor as a proof, but pretty carefully. I did not give much credit for vague discussions or examples of how to use the formula.

Use a picture to show that the level curves are tangent to each other, so that the normal vectors for the tangent planes are parallel. These are essentially the gradient vectors (see 9a) of f and g, so they are scalar multiples.