MAC 2313 Apr 3, 2019
Exam III Prof. S. Hudson

1) Let $f(x,y) = y - x$. Find the gradient of f at the point $P(2,1)$. Then sketch the gradient together with the level curve of f that passes through P.

2) Let $f(x,y) = 2x^2 - 4x + y^2 - 4y + 1$ on the closed triangle T bounded by the lines $x = 0$, $y = 2$ and $y = 2x$. Find the abs max and the abs min of f on T.

3) Given $\int_0^1 \int_y^{\sqrt{y}} \, dx\, dy$, sketch the region of integration and then reverse the order (write out an equivalent $dy\, dx$ integral, but do not evaluate it).

4) Let σ be the portion of the cylinder $y^2 + z^2 = 9$ that is above the rectangle R where $0 \le x \le 2$ and $-3 \le y \le 3$. Express the area of this surface as an iterated double integral and use that to find the surface area.

5) Use spherical coordinates to evaluate the integral. A sketch is suggested.

$$\int_{-2}^{2} \int_{\sqrt{4-x^2}}^{-\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2-y^2}} z^2 \sqrt{x^2+y^2+z^2}\,dz\; dy\; dx.$$

6) Sketch the triangular region R in the u,v plane bounded by the lines $u = 0$, $v = 0$ and $u + v = 1$. Sketch (and label) the image of R under the transformation T given by $x = 3u + 4v$ and $y = 4u$.

7) Let σ be the lamina bounded by the x-axis, the line $x = 1$ and the curve $y = \sqrt{x}$. Assume the density is $\delta(x,y) = x + y$. Find the mass of σ.

8) [20pts] True-False. Below, $f(x,y)$ is a differentiable function defined on the entire plane.

If $\nabla f(0,0) = \langle 3,4 \rangle$, then there is exactly one unit vector $\mathbf{u}$ for which $D_{\mathbf{u}} f(0,0) = 1$.

If (a, b) is a local minimum of f, then $D_{\mathbf{u}} f(a,b) = 0$ for any unit vector $\mathbf{u}$.

The function $f(x,y)$ must have an absolute maximum in the region $x^2 + y^2 < 1$.

The function $f(x,y)$ has at least one critical point on the square $[0,1] \times [0,1]$.

For any region D in the plane, $\int\int_D \, dA \ge 0$.

If f is continuous on $[a,b] \times [c,d]$, then $\int_a^b \int_c^d f(x,y) dy dx = \int_c^d \int_a^b f(x,y) dx dy$.

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It is always the case that $\int_0^1 \int_0^x f(x,y) dy dx = \int_0^1 \int_0^y f(x,y) dx dy$.

$\int_{-1}^1 \int_0^{\sqrt{1-x^2}} dy dx = \int_0^\pi \int_0^1 r dr d\theta = 2 \int_0^{\pi/2} \int_0^1 r dr d\theta$

If S is a triangle, its centroid must lie in S.

9) Choose ONE, circle it and do it. You may use the back of the page, but if so, leave a note here.

a) Prove Thm 13.7.2 as done in the text: if f is differentiable at (x_0, y_0) then the tangent plane to the graph $z = f$ at (x_0, y_0) is

$$z = f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

b) State and prove the main theorem about Lagrange multipliers, Thm.13.9.3. Your answer does not have to be quite as precise as usual, but explain the formula in detail, probably including pictures, as done in the text.

Bonus) [about 5 points] If you looked at Fubini's counterexample, describe it briefly (enough to convince me you looked at it).

Remarks and Answers: The average among the top 18 was approx 58, which is fairly normal, but a bit low. The two best scores were 91 and 76. The average results were similar on all the problems except on #6 (40%) and #3 (80%).

I took the first 3 problems from the MAC 2313 textbook to be used this summer (Thomas, 14th Edn) and already in use at FIU for MAC 2311 and 2312. You can probably borrow a copy easily for more detailed answers. See problems 14.5.1, 14.7.31 and 15.2.35. The next four are from our book, exercise 14.4.1, Example 14.6.4, ex 14.7.18 and ex 14.8.1. The first 9 TF are from http://www.mathcs.emory.edu/ heleni/m211/m211sample.pdf, which also explains the answers. The average TF score was approx 65%, which is probably a little higher than past averages.

Here is an advisory scale for Exam III. You can also use this for your current semester average (of your 3 exam scores). I do not have complete data on the HW scores. I expect those will raise the average a couple of points, but the final may bring it back down. It seems the average score on Exam I has come down due to unexpected drops, but I haven't gone back to adjust that advisory scale.

A's 65 to 100
B's 55 to 64
C's 45 to 54
D's 35 to 44

1) $\nabla f(2, 1) = \langle -1, 1\rangle$. Since $f(2, 1) = -1$, the level curve is the straight line $y - x = -1$, which should be easy to graph. After sketching in $\langle -1, 1\rangle$, you can see that the vector and the line are perpendicular, which is the main point of the exercise.

2) Abs.min = -5 at (1,2). Abs.max = 1 at (0,0). The full solution is rather long, but should include a picture and 4 stages (3 boundary lines plus the interior). One stage (for example) is the top edge where $y = 2$ and $0 \leq x \leq 1$. $f(x, y) = f(x, 2) = 2x^2 - 4x - 3$. The max of this at $x = 0$ $(f = -3)$. The min of this at $x = 1$ $(f = -5)$. After the other stages, the $f = -3$ is discarded, but the $f = -5$ is kept.

3) $\int_0^1 \int_{x^2}^x \; dy\, dx$. The region is above the parabola $y = x^2$ and below the line $y = x$. A common mistake was $\int_x^{x^2}$. Remember that $x^2 < x$ when $0 < x < 1$.

4) A fast method is to treat the surface as a (bent) rectangle, $A = bh = 2[C/2] = 2\pi r = 6\pi$. But to follow the instructions, use $SA = \int_0^2 \int_{-3}^3 \sqrt{1 + (-y/z)^2}\ dy\ dx = \cdots = 6\pi$. The calculations are a little longer than average, but standard from MAC 2312. I replaced the z, simplified a bit, and used a trig substitution to deal with the square root.

5) $2^6\pi/9$. See Ex 14.6.4 (or the lecture notes) for the full solution.

6) R has vertices at the points $A(0,0)$ and $B(1,0)$ and $C(0,1)$. Most people drew it correctly.

The image $T(R)$ is also a triangle, with vertices at the points $T(A) = D(0,0)$ and $T(B) = E(3,4)$ and $T(C) = F(4,0)$. Most people did not do well on this part. If you did not guess that $T(R)$ is a triangle (because the equations in T are linear), you might need a slower method such as this:

Alt.soln: One edge of R is where $u = 0$. When $u = 0$ we get $x = 4v$ and $y = 0$ (with $0 \le v \le 1$). These are PE's for the line segment $\overline{DF}$, which is one edge of the image. You can compute the other 2 edges this way and confirm that the result is a triangle.

7) $M = \int_0^1 \int_0^{\sqrt{x}} x + y\ dy\ dx = \cdots = 13/20$. Or, use $\int_0^1 \int_{y^2}^1 x + y\ dx\ dy$. Either way, a picture helps set this up.

8) FTFFT TFFTT

9) See the text or lectures. Most people chose b.