MAC 2313 Apr 7, 2021
Exam III Key Prof. S. Hudson

The problems are 10 points each unless labeled otherwise.

1) Write an iterated integral for $\int\int_R \, dA$, where R is bounded by $y = 3 - 2x$, $y = x$ and $x = 0$. You don't have to evaluate it.

2) Use a double integral to find the area enclosed by $r = 2 + \cos\theta$. For partial credit you can use Calc 2 methods instead.

3) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} (x^2 + y^2) dy\, dx$ by converting to polar coordinates.

4) Find the mass of the solid G. It is bounded by $0 \le x \le 1$, $0 \le y \le 2$, $0 \le z \le y$ with density $\delta(x, y, z) = x$.

5) Evaluate $\int\int\int_E |xyz| dV$, where E is the ellipsoid $x^2 + (y/2)^2 + (z/3)^2 \le 1$. Hint: Let $x = u$, $y = 2v$ and $z = 3w$. Then integrate over a region in uvw space, including a Jacobian.

6) Evaluate the line integral $\int_C y dx + z dy - x dz$ along the curve $x = t$, $y = t^2$, $z = t^3$ from $(0, 0, 0)$ to $(1, 1, 1)$.

7) Let $\mathbf{F}(x, y) = \langle 6xy, 3x^2 \rangle$. Show that this is a conservative vector field. Find a potential function f for it.

8) [5 pts] Suppose $f(x, y, z) = x^2 + yz$ and $\mathbf{F} = \nabla f$. Let C be the curve $\mathbf{r}(t) = \langle \sin(\pi t), \cos^2(\pi t), t + 1 \rangle$ for $0 \le t \le 2$. Find the work done by $\mathbf{F}$ on C. This is meant to be fairly quick and easy.

9) [15pts] True-False.

If S lies in the first octant of R^3 then the centroid of S must also lie in the first octant.

The centroid of the region in R^2 bounded by $r = 1 + \cos\theta$ lies on the $y-$axis.

The average value of $f(x, y) = xy^2$ over $0 \le x \le 1$, $-1 \le y \le 1$, is zero.

$\int_0^1 \int_0^1 \int_0^z \, dx\, dy\, dz = \int_0^1 \int_0^1 \int_0^x \, dz\, dy\, dx$

$\int_0^\pi \int_0^{\pi/2} \int_0^1 \rho \, d\rho\, d\theta\, d\phi = 2 \int_0^{\pi/2} \int_0^{\pi/2} \int_0^1 \rho \, d\rho\, d\theta\, d\phi$

10) Choose ONE to prove:

a) State and prove Eqn (3) of Ch.14.6, page 855, which is the equation of the tangent plane to the graph of $z = f(x, y)$ at a point P_0 on the graph. You can use Eqn (1) of Ch.14.6, which is the equation of the tangent plane to a level surface $F(x, y, z) = c$ at a point P_0 on it.

b) Explain the $r dr d\theta$ appearing in polar integrals coming from rectangular ones. Use pictures and geometric formulas rather than a determinant.

Remarks: The average was 61 without many extremely high or low scores. The top 2 were 78 and 75. The average score on most problems was good, but not problems 5, 7 and 8 (25% combined average). The scale for your current semester average is about the same as the scale on the Exam 2 Key (A's starting at 71, etc). The scale for Exam 3 is similar, but 2 points lower (A's starting at 69, etc). I expect the final semester scale will also be similar, assuming normal results on the final exam and the HW.

1) $\int_0^1 \int_x^{3-2x} dy\ dx$. Draw a picture. R is the triangle beside the y-axis.

2) Draw a picture. The region looks like a deformed circle centered near the origin. $\int_0^{2\pi} \int_0^{2+\cos\theta} r\ dr\ d\theta = \cdots = \frac{9\pi}{2}$.

3) Draw a picture. It is a quarter-circle. $\int_0^{\pi/2} \int_0^1 r^2\ r\ dr\ d\theta = \pi/8$.

4) 1

5) 6. Very few people did all aspects of this one correctly. You will integrate over the sphere $u^2 + v^2 + w^2 \leq 1$, which is much easier in spherical coordinates ($\rho \leq 1$). This is the main reason to do a substitution here. But restrict to the 1st octant (using symmetry) in order to remove the absolute values. Get $J = 6$. Replace the xyz below by $6uvw$ and u by $\rho \sin\phi \cos\theta$, etc. This is 15.8.19. We did it in class (I think).

$$8 \int_0^{\pi/2} \int_0^{\pi/2} \int_0^1 xyz\ 6\ \rho^2 \sin\phi\ d\rho\ d\phi\ d\theta = \cdots = 6$$

6) -1/60. $\int_C y dx = \int_0^1 (t^2)(1)\ dt = 1/3$, etc. $\quad 1/3 + 2/5 - 3/4 = -1/60$.

7) It is conservative because $M_y = N_x = 6x$. The potential is $f = 3x^2 y$ (but include some work such as $\int M\ dx = 3x^2 y + C(y)$, etc).

8) The simple FTLI answer is $f|_A^B$, with $A = \mathbf{r}(0) = \langle 0, 1, 1\rangle$, and $B = \mathbf{r}(2) = \langle 0, 1, 3\rangle$. So, $3 - 1 = 2$.

9) TFFTT

10) See the text or lectures.