1) Find parametric equations for the line $L$ that passes through the points $(1, 2, 2)$ and $(3, -1, 3)$.

2) Find the equation of the plane tangent to $z = x^3 - y^3$ at $(3, 2, 19)$.

3) Find $\partial w/\partial s$, given that $w = \ln(x^2 + y^2 + z^2)$, $x = s - t$, $y = s + t$, and $z = 2\sqrt{st}$.

4) Sketch the region of integration, reverse the order of integration, and evaluate the resulting integral:
$$\int_0^\pi \int_0^\pi \frac{\sin y}{y} \, dy \, dx$$

5) The plane $x + y + z = 12$ intersects the paraboloid $z = x^2 + y^2$ in an ellipse. Use LaGrange multipliers to find the highest and lowest points on this ellipse.

6) Let $u = xy$ and $v = y/x$. Solve for $x$ and $y$ in terms of $u$ and $v$. Then compute the Jacobian $\partial(x, y)/\partial(u, v)$.

7) Find a potential function for $\mathbf{F}(x, y) = \langle 3x^2 + 2y^2, 4xy + 6y^2 \rangle$ (or explain why this is not possible).

8) Apply Stoke’s Thm to evaluate $\int_C \mathbf{F} \cdot \mathbf{T} \, ds$ where $C$ is the ellipse in which the plane $z = y + 3$ intersects the cylinder $x^2 + y^2 = 1$. Orient the ellipse counterclockwise as viewed from above and take $\mathbf{F}(x, y, z) = \langle 3z, 5x, -2y \rangle$.

9) Answer True or False.

- If $(a, b, c)$ is a critical point of $f$ then $f_x(a, b, c) = 0$.
- The origin is a saddle point of $z = y^2 - x^2$.
- If $R$ is a parallelogram with constant density, then its centroid is a point in $R$.
- The region where $1 < r < 2$ (in polar coordinates) is simply-connected.
- $\mathbf{F}(x, y) = \langle -y, x \rangle$ is a conservative vector field.

10) Choose ONE:

- A) State and prove Green’s Thm
- B) State and prove the Fundamental Theorem of Calculus for Line Integrals.
**Answers and Remarks:** The average score was 63/100, based on the best 26 out of 33 scores. Grades were generally high on problems 3, 4, and 7, but low on problems 5 and 8. Problems 1 and 2 should have been easy.

1) The vector from P to Q is \(\langle 2, -3, 1 \rangle\). So, \(x = 1 + 2t, y = 2 - 3t\) and \(z = 2 + t\) (but other answers are possible).

2) The normal vector is \(n = \langle 27, -12, -1 \rangle\), so \(27(x - 3) - 12(y - 2) - (z - 19) = 0\)

3) \(2(x + y + z) \sqrt{t/s} / (x^2 + y^2 + z^2)\) which can be simplified to \(2/(s + t)\) (optional). I gave full credit for correct answers from any method. When there were errors, I gave more partial credit if you used the multivariable chain rule (see 12.7-5).

4) 2. This was on a previous exam.

5) Set \(f(x, y, z) = z\). Min at (2,2,8). Max at (-3,-3,18). See 12.9 Example 4. I did not give partial credit unless you chose \(f\) correctly, or at least found a point satisfying both constraints.

6) \(y = \sqrt{uv}, x = \sqrt{u/v} \) (a ± sign is optional; it doesn’t affect \(J\)). Then \(J = 1/(2v) = x/(2y)\). See 13.9-3.

7) \(\phi(x, y) = x^3 + 2xy^2 + 2y^3\).

8) This is Ch 14.7, Example 1. \(\int \int_S \text{curl} F \cdot n \ dS = \int \int_S \sqrt{2} \ dS = 2\pi\) (include more work than this, of course).

9) FTTFF

10) See the text. I graded these proofs mostly by checking whether you included the key ideas. For example, in A) I really wanted to see the phrases *vertically simple*, and *by the Fundamental Theorem of Calculus*. For full credit, also explain the P and Q split, the minus sign, and why the sides drop out. Also suggested: draw a picture, and mention more general regions.