MAC 2313 Apr 27, 2017
Final Exam Prof. S. Hudson

1) Find all critical points of the function $f(x, y) = xy - x^3 - y^2$. Classify the critical points as either relative maxima, relative minima or saddle point.

2) [7 pts each] 2a) Let L be the line defined by the parametric equations $x = 1 - 2t, y = 2 - 3t, z = 3 + t$. Let P be the plane defined by $3x + 2y + z = 0$. Show that L and P are NOT perpendicular to each other.

b) Find an equation for the plane Q that both contains L and is perpendicular to P.

3) [7 points] Find the normal scalar component of acceleration when $t = 1$, given that $\mathbf{a}(1) = \mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ and $a_T(1) = 1$.

4) [7 points] Find the distance from the point $P(1, 1) \in R^2$ to the line $y = 3x + 1$. Hint: find a normal vector to the line, and then a projection.

5) Evaluate the surface integral $\int\int_\sigma z^2 dS$, where σ is the portion of the cone $z = \sqrt{x^2 + y^2}$ between $z = 1$ and $z = 2$.

In problems 6 and 7, let G be the solid bounded above by the sphere $\rho \leq 4$ and below by the cone $\phi = \pi/3$.

6) [7 points] Use spherical coordinates to find the volume of G.

7) [7 points] Express $\int\int\int_G x^2 + y^2 \; dV$ as an iterated integral in cylindrical coordinates. You do not have to compute it.

8) [18 pts] True - False:

If $\mathbf{F}$ is an inverse-square field on a solid $G \subset R^3$, then the flux across the boundary is 0.

If D is an open set in R^2 then every point of D is an interior point of D.

The Stoke's Theorem equates a triple integral with a surface integral.

If $f_y = g_x$ on R, an annulus in R^2, then $\mathbf{F} = \langle f, g \rangle$ is a conservative vector field on R.

If $f(x, y)$ is continuous on the disk $x^2 + y^2 \leq 1$, and is not constant, then it has a maximum value on the disk.

The Jacobian of the transformation $u = x + 5y$, $v = 5x - y$, is a constant.

9) Use the Divergence theorem to compute the flux of $\mathbf{F}$ across σ with outward orientation, given that σ is the surface of the solid box bounded by the coordinate planes and $x = 4$, $y = 3$ and $z = 2$, and

$$\mathbf{F} = \langle x^2 + y, z^2, e^y - z \rangle$$

10) Choose ONE proof:

a) [Thm 13.6.6] ∇f at P is normal to the level curve of f through P. You can give a careful explanation, as in the text, rather than a formal proof.

b) State and prove the formulas for a_N and a_T in Thm.12.6.3, which are given in terms of $\mathbf{v}$ and $\mathbf{a}$. You may include a picture, but also explain the ideas in words.

Remarks and Answers: The average was 60 / 100, with the 4 highest scores approx 83. The lowest scores were on problems 2b, 3, 4 and 7 (averaging approx 36%) and the best were on problems 5 and 6 (averaging approx 83%). Most people did better on the more recent material than on the review problems. As usual, I do not directly scale the final exam, and have not yet curved the semester grades.

1) Set $0 = \nabla f(x, y) = \langle y - 3x^2,\ x - 2y\rangle$ to get critical pts $(0, 0)$ and $(1/6, 1/12)$. Using D, the first is a saddle point and the other is a rel. max.

2a) The direction of the line is $\mathbf{v} = \langle -2, -3, 1\rangle$. A normal vector to the plane is $\mathbf{n} = \langle 3, 2, 1\rangle$. They are not perpendicular, because their dot product is not zero.

Many people confused perpendicular with parallel, possibly because we did a similar review problem with 'not parallel'. This was an unfortunate coincidence, so I graded gently for this kind of mistake. To show they are not parallel we could check they are not scalar multiples of each other, or compute the cross product.

2b) A normal vector to the new plane is $\mathbf{v} \times \mathbf{n} = \langle -5, 5, 5\rangle$, so

$$-5(x - 1) + 5(y - 2) + 5(z - 3) = 0$$

3) $\sqrt{8}$. See Exam 2 problem 6.

4) $\frac{3}{\sqrt{10}}$. See Exam 1 problem 3.

5) Using x, y as parameters, get $dS = \sqrt{2}\ dA$ and then replace dA by $r\ dr\ d\theta$, z^2 by r^2 to get

$$A = \sqrt{2} \int_0^{2\pi} \int_1^2 r^3\ dr\ d\theta = \frac{15\pi\sqrt{2}}{2}$$

6) $\int_0^{2\pi} \int_0^{\pi/3} \int_0^4 \rho^2 \sin\phi\ d\rho\ d\phi\ d\theta = 64\pi/3$.

7) $\int_0^{2\pi} \int_0^{2\sqrt{3}} \int_{\frac{r}{\sqrt{3}}}^{\sqrt{16-r^2}} r^3\ dz\ dr\ d\theta$. See Ch14 Review, problem 23. To find the limits, draw a picture and find (from trig) that $(2\sqrt{3}, 0, 2)$ lies on the intersection of the sphere and the cone. And that $r = x$, $z = r/\sqrt{3} = \sqrt{16 - r^2}$ there.

8) FTFFTT

9) $\Phi = \int\int\int_G 2x - 1 \; dV = 72.$

10) See the text.