MAC 2313 Sept 3, 2020
Quiz I Key Prof. S. Hudson

1) (25pts) Give a geometric description of the set of points in space whose coordinates satisfy the pair of equations: $x^2 + z^2 = 4$ and $y = 0$. For example (but not correct) "It is a sphere of radius 3 in the first octant, centered at (4,5,6)".

2) (30pts) Sketch the curve $r = 4 + 3\cos\theta$ including at least two labeled points. Calculate the area enclosed by this curve.

3) (45pts) 3a) Let $\mathbf{F} = \langle 2, 3\rangle$ and $\mathbf{u} = \langle 4, 1\rangle$. Compute the vector projection of $\mathbf{F}$ onto $\mathbf{u}$ (Compute $\mathbf{p} = \text{proj}_{\mathbf{u}}\mathbf{F}$).

3b) Show with a calculation that $\mathbf{v} = \langle -2, 8\rangle$ is orthogonal to $\mathbf{u}$. Explain your reasoning.

3c) Find the vector components of $\mathbf{F}$ in the directions of $\mathbf{u}$ and $\mathbf{v}$. Hint: this is related to 3a and/or 3b.

Remarks and Answers: The average grade was approx 50 (not including zeroes and some other very low scores) with high scores of 80 and 76. This is quite low and it appears some people have barely started studying yet. I do not set an official scale for minor quizzes, but estimate that a 60 would be a B-.

1) It is a circle of radius 2 in the xz-plane, centered at (0,0,0).

It is OK to say more, but this description is enough because there is only one circle that fits all the info. It is not OK to say less, but you probably got partial credit. It is not OK to call it a sphere. The circle is not in the first octant because it includes points like (-2,0,0).

For more practice, see the first exercises in HW 1.

2) The curve is a dimpled limacon. It looks a bit like a circle but it has a dimple (a dent) on the left side. It passes thru (7,0) and thru (0,4), [from setting $\theta = 0$ then $\pi/2$], etc. See the answer key to exercise 11.4.27b for a similar curve, but rotated 90 degrees. The calculation goes

$$A = \frac{1}{2}\int_0^{2\pi} (4 + 3\cos\theta)^2 \, d\theta = \cdots = (16 + \ 9/2)\pi = 41\pi/2.$$

You should work out the integral for practice, since this type appears often in MAC 2313. I used some shortcuts from class, such as $\int_0^{2\pi} \cos x \, dx = 0$ and $\int_0^{2\pi} \cos^2 x \, dx = \pi$.

For more practice, see the exercises in Ch.11.5 (and a review problem done on day 1).

3a) Use $\frac{\mathbf{F}\cdot\mathbf{u}}{||\mathbf{u}||^2}\mathbf{u} = \frac{11}{17}\langle 4, 1\rangle$.

3b) Since $\mathbf{v} \cdot \mathbf{u} = -8 + 8 = 0$ the vectors are orthogonal. I did not give full credit for "swapping the components" remarks (this is not really a calculation and is not very general - it doesn't work in R^3).

3c) The component in the **u** direction is the projection computed in 3a. The component in the **v** direction is a similar projection, $\frac{1}{17}\langle -10, 40\rangle$.

You can check that **F** is the sum of its components, $\mathbf{F} = \frac{11}{17}\langle 4, 1\rangle + \frac{1}{17}\langle -10, 40\rangle$. Or you could use this principle as a shortcut for 3c. Components are used in basic physics problems such as Ex.9 on page 725. The solution on page 725 doesn't use the usual formula for a vector projection because that comes later in the book. They do compute the projections, but they use trig.

For more practice, see the first exercises in Ch.12.3 and examples from class.