MAC 2313 Feb 1, 2021
Quiz 1 Key Prof. S. Hudson

1a) Given points $P_1(3,4,5)$ and $P_2(2,3,4)$, find the direction (a unit vector) of $\overrightarrow{P_1P_2}$.

1b) Find the midpoint of the line segment $\overline{P_1P_2}$.

2) Suppose $\mathbf{u}_1$ and $\mathbf{u}_2$ are orthogonal unit vectors in R^3 and that $\mathbf{v} = 4\mathbf{u}_1 + 5\mathbf{u}_2$. Find $||\mathbf{v}||$ showing all work and explaining any key steps. For max credit use a dot product or cross product etc.

3) Find parametric equations for the line L through $(-2,0,4)$ parallel to $\mathbf{v} = 2\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$.

Remarks, Answers: The problems were worth 40, 30 and 30 points, resp. Probably problem 2 was hardest, but I don't have stats on that. The top two scores were both 93. Eight people scored above 50. The average among the top 10 was 64. While this quiz only counts for approx 2% of your grade, a very low score indicates you are not learning enough from the homework. You should probably see me or Melissa about better study habits before Exam I. A rough scale for the quiz is

A's 72 - 100
B's 62 - 71
C's 52 - 61
D's 42 - 51

1a) $\overrightarrow{P_1P_2} = \langle 2,3,4\rangle - \langle 3,4,5\rangle = \langle -1,-1,-1\rangle$. It has norm $\sqrt{3}$ so it is not a unit vector and must be normalized. The direction vector is $\mathbf{u} = \frac{1}{\sqrt{3}}\langle -1,-1,-1\rangle$.

1b) $(2.5, 3.5, 4.5)$. The simplest calculation is to average the given components, $(2+3)/2 = 2.5$, etc. Other methods are OK. For example, to go halfway from P_1 to P_2 add $\langle 3,4,5\rangle + \frac{1}{2}\langle -1,-1,-1\rangle$ to get the same answer (but change the brackets to parentheses).

This problem doesn't involve involve triangles or any more complicated shapes, so we don't need tools like the dot product here. This is exercise 12.2.37.

2) $\sqrt{41}$, based on $||\mathbf{v}||^2 = \mathbf{v}\cdot\mathbf{v} = (4\mathbf{u}_1 + 5\mathbf{u}_2)\cdot(4\mathbf{u}_1 + 5\mathbf{u}_2) = 16 + 25$. In the last step, I used $\mathbf{u}_1\cdot\mathbf{u}_1 = ||u_1||^2 = 1$ and $\mathbf{u}_1\cdot\mathbf{u}_2 = 0$ (etc) to simplify.

We used this method in the proof of the $\mathbf{u}\cdot\mathbf{v}$ theorem and on problem 3 of the first worksheet. I gave some credit for using a picture of a right triangle and the Pythagoras theorem (though the setting here is R^3 not R^2), but the dot product is a better general purpose method for most geometric questions like this one. This is a variation on exercise 12.3.29.

3) I'd start with the standard vector equation $\mathbf{r}(t) = \langle -2,0,4\rangle + t\langle 2,4,-2\rangle$. Then, put that into P.Eqn form, $x = -2 + 2t$, $y = 4t$ and $z = 4 - 2t$. This is Example 1 from Ch. 12.5.