MAC 2313 Mar 1, 2021

Quiz 2 Prof. S. Hudson

1) Given a curve $\mathbf{r}(t) = \langle 3\cos t, 3\sin t, 4t\rangle$, find the equation of the osculating plane when $t = 0$. Your work should include these formulas with justification (show enough work): $\mathbf{T}(t) = \frac{1}{5}\langle -3\sin t, 3\cos t, 4\rangle$ $\mathbf{N}(0) = \langle -1, 0, 0\rangle$

2) Show that $\lim_{(x,y)\to(1,1)} \frac{xy^2-1}{y-1}$ does not exist, by considering the limits along two different paths (such as $x = y$ and $x = 1$). Include a sentence at the end to explain your reasoning. You can use the algebra formula $a^3 - 1 = (a^2 + a + 1)(a - 1)$.

3) Find dw/dt given that $w = \frac{x}{z} + \frac{y}{z}$, $x = \cos t$, $y = 2t$ and $z = 1/t$. For full credit, use the Chain Rule, though you may check your answer the other way, by expressing w directly as a function of t.

Remarks, Answers: The average score was approx the same as Exam I, approx 66. The 4 highest scores were in the high 80's. You can use the Exam I scale for this quiz. The problems were worth 30, 40 and 30 resp. They were taken (or modified) from exercises 13.5.7, 13.5.9, 14.2.49 and 14.4.3.

1) For 5 points, I reminded some people that the osculating plane contains the point $\mathbf{r}(0)$ and the vectors $\mathbf{T}(0)$ and $\mathbf{N}(0)$. You can deduce that the normal vector is $\mathbf{n} = \mathbf{B}(0) = \langle 0, -4/5, 3/5\rangle$ so the equation is $(-4/5)(y-0)+(3/5)(z-0) = 0$ which simplifies (optional) to $3z - 4y = 0$.

Some people who started well seemed to forget what the equation of a plane should look like. Several answers looked like the vector equation of a line, etc.

2) On the path $y = x$ the limit simplifies to $\lim_{x\to1} x^2 + x + 1 = 3$. On the path $x = 1$ it simplifies to $\lim_{y\to1} y + 1 = 2$. Since these differ, the original limit does not exist. The answers were mostly good on this one.

3) Using the Chain Rule ($\frac{dw}{dt} = w_x \frac{dx}{dt} +$ etc), get $-t\sin t + 2t + (\cos t + 2t)$. Remember to convert all variables into t notation at the end. The other method starts with $w(t) = t\cos t + 2t^2$ which leads to the same answer.