MAC 2313 Oct 15, 2020

Quiz 3 Key Prof. S. Hudson

1) (30pts) Let $\mathbf{r}(t) = (t+1)\mathbf{i} + 2t\mathbf{j} + t^2\mathbf{k}$. Write $\mathbf{a}$ in the form $\mathbf{a} = a_T\mathbf{T} + a_N\mathbf{N}$ at $t = 1$ without finding $\mathbf{T}$ and $\mathbf{N}$.

2) (30pts) Calculate all four second-order partial derivatives of $g(x, y) = x^2 y \cos y + y \sin x$.

3) (30pts) Let $f(x, y, z) = x/y - yz$. Find the direction in which f increases most rapidly at the point $P(4, 1, 1)$.

Remarks + Scale: The problems came from 13.5.3, 14.3.43 and 14.5.21. By accident, the quiz was out of 90 points, so I will multiply each score by 100/90 to correct for that. The average was approx 55 out of 90, with high scores of 80 and 75. An advisory scale for the quiz:

A's 63 - 90
B's 53 - 62
C's 43 - 52
D's 33 - 42

Answers:

1) One fairly short way to compute the two coefficients is this:
$\mathbf{r}'(t) = \mathbf{i} + 2\mathbf{j} + 2t\mathbf{k}$.
$\mathbf{a} = \mathbf{r}''(t) = 2\mathbf{k}$ and $||\mathbf{a}|| = 2$.
$ds/dt = ||\mathbf{r}'|| = (5 + 4t^2)^{1/2}$
$a_T = d^2s/dt^2 = 4t(5 + 4t^2)^{-1/2} = 4/3$ when $t = 1$.
$a_N = \sqrt{||\mathbf{a}||^2 - a_T^2} = 2\sqrt{5}/3$.

Answer: $\mathbf{a} = 4/3\mathbf{T} + 2\sqrt{5}/3\mathbf{N}$.

2) $g = x^2 y \cos y + y \sin x$, $g_x = 2xy \cos y + y \cos x$, $g_y = x^2[\cos y - y \sin y] + \sin x$. None of these are part of the final answer, since these are not second order. Answers:

$g_{xx} = 2y \cos y - y \sin x$.

$g_{xy} = 2x(\cos y - y \sin y) + \cos x$.

$g_{yx} = 2x(\cos y - y \sin y) + \cos x$. You might save time using $g_{yx} = g_{xy}$ but technically you should check that both are continuous. Also computing both is a good way to check your work.

$g_{yy} = x^2(-2 \sin y - y \cos x)$.

3) The direction is given by the gradient vector; $\nabla f(x, y, z) = \langle 1/y, -x/y^2 - z, -y\rangle = \langle 1, -5, -1\rangle$. You can stop here for full credit, but it might be even better to normalize.

If you did not circle your answer, and your work ended with a scalar (such as the maximal derivative), I could not give full credit.