MAC 2313 Mar 29, 2021
Quiz 3 Key Prof. S. Hudson

1) Find the average height of the cone $z = \sqrt{x^2 + y^2}$ over the region where $x^2 + y^2 \leq 4$. Suggestion: polar coordinates.

2) G is the solid with base $z = 0$ and top $z = y^2$, with $0 \leq x \leq 1$ and $-1 \leq y \leq 1$. Compute the volume of G using a triple integral in rectangular coordinates.

3) D has a base where $z = 0$, the sides are the cylinder $r = \sin\theta$, and the top is $z = 3r^2$. Find the 6 limits of integration in cylindrical coordinates for the triple integral of f over D (so, fill in the blanks below, but do not try to evaluate this).

$$\int \int \int f(r, \theta, z)\ dz\ r\ dr\ d\theta$$

Remarks: The problems were worth 30, 40 and 30 points. 3a and 3b were worth 6 and 24. These were taken from exercises 15.4.34, 15.5.23 and 15.7.35.

The 2 highest scores were 91 and 90. The average was approx 68 out of 100, which is a little higher than the semester average. But I don't think it will affect the semester scale much at all. Here is an advisory scale for the Quiz:

A's 76 to 100
B's 66 to 75
C's 56 to 65
D's 46 to 55

1) 4/3. In general, you can compute an average by dividing (like (3+5+7)/3 = 5). Here, $\int_0^{2\pi} \int_0^2 r\ r\ dr\ d\theta = \cdots = 16\pi/3$, divided by the area, 4π, gives 4/3.

2) $V = \int_0^1 \int_{-1}^1 \int_0^{y^2} 1\ dz\ dy\ dx = \cdots = 2/3$.

3a) The results were poor, so I went over this one in class on 3/31/21. It is a circle with radius 1/2 centered at (0,1/2) passing through P(0,1) for example. Note that P is where $\theta = \pi/2$ and $r = \sin(\pi/2) = 1$. With practice, you can plot several points like P this way and connect the dots. The important thing for 3b is that $0 \leq \theta \leq \pi$.

3b) $\int_0^{\pi} \int_0^{\sin\theta} \int_0^{3r^2} f(r, \theta, z)\ dz\ r\ dr\ d\theta$. This was easy and the results were fairly good, except maybe for $\theta \leq \pi$.