MAC 2313 Nov 10, 2020
Quiz 4 Prof. S. Hudson

1) (40pts, changed to 55pts) Sketch the region bounded by the lines $y = x$ $y = x/3$ and $y = 2$. Then express its area as a double integral and evaluate that.

2) (30pts, changed to extra credit) Evaluate the integral $\int_0^\pi \int_0^\pi \int_0^{2\sin\phi} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$.

3) Option A (30 pts max, changed to 45): Use a 3x3 determinant to find the Jacobian of the transformation T from cylindrical to rectangular coordinates given by $x = r\cos\theta$, $y = r\sin\theta$, $z = z$.

3) Option B (25 pts max, changed to 40): this is slightly easier, but is worth at most 25 points. Don't do both. If you choose this one, circle the words *Option B.*

Use a 2x2 determinant to find the Jacobian of the transformation T from polar to rectangular coordinates given by $x = r\cos\theta$, $y = r\sin\theta$.

Remarks, Answers: The average was 83 out of 100. But up to 30 more extra credit points were possible on problem 2. There were three high scores of 125.

1) The best way is $\int_0^2 \int_y^{3y} \, dx \, dy = 4$. I also gave credit if you started from $\int_0^2 \int_{x/3}^{x} \, dy \, dx + \int_2^4 \int_{x/3}^{2} \, dy \, dx$. You can check your answer using $A = bh/2 = 4$.

2) This is problem 15.7.43. I decided it was too hard while proofreading the quiz, and meant to change it to 15.7.47. Apparently I forgot to do so. I agreed to count this problem as extra credit. I gave 15 points for getting as far as $\int_0^\pi \int_0^\pi \frac{8}{3} \sin^4\phi \, d\phi \, d\theta$. Three people continued successfully to the correct answer π^2 and got 30 points.

3a) $J = \det \begin{pmatrix} \cos\theta & -r\sin\theta & 0 \\ \sin\theta & r\cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} = r$

3b) $J = \det \begin{pmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{pmatrix} = r$