MAD 4302 Dec 5, 2013
Outline Prof. S. Hudson

John asked me to outline the course, as I see it; this page is intended to do that. It is not really intended as a review sheet for the final, but I'll assume that was part of the reason for the request. I have also included the Pirate puzzle, and another one near the end. I have the answers somewhere, so email me if interested.

First, this course develops thinking skills useful in programming, probability, and many branches of math and science, mainly through diverse exercises which do not necessarily fall into clear and distinct categories. Thus covering chessboards with dominoes or rooks, the game of NIM, counting problems about eggs and baskets, etc, are important "experiences" rather than important "facts". Though these may appear on the final, I would not suggest cramming for such problems, from Chs 1-3 for example, in the last days of the semester. I'd suggest reviewing a few examples and the notation, such as $P(n, r)$ and $r(m, n)$ as needed.

The latter parts of the course continue the skill-building, but also include more specific problems and methods of solution. They are fairly independent of each other, and it was largely a matter of taste which topics to cover.

Ch.5: About $C(n, r)$

The basics are the binomial theorem (which we proved two ways) and Pascal's Triangle / Identity. We practiced with enough identities that you should feel competent about manipulating $C(n, r)$ as needed, but I wouldn't expect you to remember all the formulas (page 136-137 for example). We covered this chapter pretty well, and proved most of the theorems. You should be able to use or state those results.

Ch.6: Inc-Exc.

This chapter has one main idea, which you should know well by now. There is less meat here than Ch 5. You may also need to memorize a few formulas (Thm 6.3.1, 6.4.1, etc), but can skip Ch. 6.6.

Ch.7: Recurrence relations

These appear so often in combinatorics that they are worthy of study, the main goal being to 'solve' them. There is no single sure-fire method, but various ideas. Probably Ch.7.4 and 7.5 are the core sections, containing solution methods similar to differential equations, which may require some educated guesses about the form of the answer. I view generating functions as an 'alternative approach', worthy of [a bit less] study, though others may value them more highly. We covered 7.6 pretty well, as a non-linear example, in which $g(x)$ is really useful, and where the Catalan numbers arise.

Ch.8: Differences

Know how to make and use difference tables. Know the special sequences. Omit Ch 8.5. We covered most of 8.1 to 8.4, but skipped around a little bit.

Ch.9 and 13.3: SDR's

We covered this short chapter pretty thoroughly. You may need to skim over some parts of Ch 11 and 12 (mainly 12.5) for graph vocabulary, but if you followed my lectures, you are probably already OK on that. You should understand how / why the matching algorithm works and learn any proofs mentioned, for example, on my HW page.

Here are two interesting logic puzzles, mainly for fun.

1) PIRATES: Five pirates find 100 gold pieces on a deserted island. How will they split it up? According to Pirate Law, the oldest pirate (Pirate A) must propose a way to split it up (for example, I get 40 and you guys get 15 each). Then the 5 pirates vote on whether to accept that proposal or to kill Pirate A. If they decide to kill him, it is Pirate B's turn to make a proposal and they vote (in case of a tie such as 2-2, the proposal is accepted). And so on.

Now, what should Pirate A propose ? You can assume that each pirate will vote in a completely rational manner, based on these goals:

a) He wants to live. And if that doesn't decide his vote: b) He wants as much gold as possible. And if that doesn't decide his vote: c) He'd prefer the other pirates to die.

2) The Warden (Story 1; The Cell): A prison holds 23 prisoners in 23 cells. The Warden decides to give them a chance to go free. Once in a while, he will choose one prisoner randomly and take them to a special cell, which contains just one switch (perhaps a light switch - but it doesn't matter). The prisoner can flip the switch, or not, and then he leaves. The game ends when one of the prisoners declares that all 23 prisoners have been to the special room at least once. If he is right, they all go free, but if he is wrong they are all killed. No prisoner knows how many prisoners have gone into the cell before him - he knows nothing except the position of the switch when he enters. No talking, peeking, or other communication is allowed after this game starts. But the prisoners are allowed to form a plan before it starts. What's the plan?