MAD 4302 Sept 26, 2013
Quiz 2 and Key Prof. S. Hudson

1) How many ways can 2 blue rooks, 2 white rooks and 2 red rooks be placed on an 8x8 chessboard so that no two attack each other ?

2) Answer True or False to each; you do not have to explain.

$K_5 \to K_3, K_3$

$K_5 \to K_5, K_2, K_2$

$K_7 \to K_3, K_3$

$r(3, 2) = 2$

3) How many towers are there, $\emptyset \subseteq A \subseteq B \subseteq \{1,\ 2, \cdots 10\}$? (as usual, explain)

4) How many numbers in $\{1,\ 2, \cdots 10,000\}$ have their digits sum to 7 ? (ex: 502).

5) We choose 151 integers from $\{1,\ 2, \cdots 300\}$. Show that one chosen must divide some other one chosen (you can answer on the back).

Remarks and Answers: The average was 71 (among the top 15) with highs of 102 and 93. The rough scale is

A's 78 - 100
B's 68 - 77
C's 58 - 67
D's 48 - 57

1) $(8!)^2/32$. There are many ways to approach this, but one of the simplest is this sequence of decisions:
Which rows to use ? $C(8, 6)$
Which columns to use ? $C(8, 6)$
Which 6 locations to use among the remaining 36 squares? 6!
Which colors go on which squares ? 6!/8
Then, multiply these 4 numbers.

2) FTTF

3) 3^{10} (decision 1 is whether $1 \in A$ or $1 \in B - A$ or it is in neither; 3 options). There were other successful approaches, but this is simplest.

4) This is a $x_1 + x_2 + x_3 + x_4 = 7$ problem (eg 7 eggs in 4 baskets), so $C(10, 3)$.

5) [similar to HW and lecture examples, so this will be brief] Factor each chosen $n = 2^k q$, where q is odd. There are 151 n's and at most 150 q's, so some chosen pair shares a q.

The smaller of these two divides the other.

Bonus (mostly for fun)) This was to solve problem 1 with the extra rule that no rook can go into a corner. The most natural approach seems to be:

Let A be your answer to problem 1.
Let $B =$ the number of ways to do it with a rook in a corner.
Let $C =$ the number of ways to do it with 2 rooks in corners (opposite corners).

Then, Answer $= A - B + C$ by Subtraction and Inclusion-Exclusion. It seems that $C = 2C(6,2)^2 4!6!/8$ from reasoning like problem 1 (but I only spent a few minutes on this). At a glance, B seems similar.