MAD 4302 Oct 10, 2013
Quiz 3 and Key Prof. S. Hudson

1) Show that among 20 people at a party, there are two who have the same number of acquaintances (since knowing yourself does not count, your number of acquaintances could vary from 0 to 19).

2) How many paths of length 13 from Start to Finish, avoiding A, B, C and D ? For full credit, give a fairly simple answer (no more than 8 terms) with some explanation. The map is on the board; it has 8 rows and 7 columns. Start is at (1,1) and Finish is at (7,8). A is (4,4), B is (5,4), C is (4,3) and D is (5,3). *These ordered pairs have been corrected to match the map on the board during the quiz.*

3a) State Sperner's Thm.

3b) Prove it (I will give some extra credit for this), or explain what α_k and β mean, where

$$\beta = \sum_{k=0}^{n} \alpha_k k!(n-k)!$$

Remarks: The problems were worth 30, 30 and 40 points resp. The average was 70 (among the top 15) with highs of 100 and 84. The rough scale is the same as Quiz 2:

A's 78 - 100
B's 68 - 77
C's 58 - 67
D's 48 - 57

Average grade: I averaged your 3 quiz scores and estimated your current semester letter grade (not yet including HW or extra credit) in the upper left corner of your quiz. You should check this. The average of these is 78, the highest is 88 and the scale is:

A's 84 - 100
B's 76 - 83
C's 66 - 75
D's 56 - 65

Answers:

1) [briefly] If someone knows everyone, then the number of acquaintances for each person varies from 1 to 19 (these numbers are the 19 pigeonholes). Since $20 > 19$ the PHP implies two people share the same number. On the other hand, if nobody knows everybody, the number varies from from 0 to 18. Again there are only 19 pigeonholes, and similar reasoning applies.

2) There are $C(13,6)$ ways all together (ignoring the forbidden positions). Observe that any path through ABCD goes through A or D, but not both, so B and C can be ignored. The number through A is $C(6,3) \cdot C(7,3)$ and D is similar. By the substraction rule:

$$C(13,6) - C(6,3) \cdot C(7,3) - C(6,2) \cdot C(7,2) = 701$$

Two people got this by recursion (by filling in each blank cell with a number, working from F towards S).

Most people tried something else and got bogged down, but got some partial credit based on their main idea, clarity and calculations. Inclusion-Exclusion is possible (if you didn't notice that B and C can be ignored), but it gets rather messy.

3a) [20 pts] See the text, Thm 5.3.3. It is about subsets. Several people confused this with similar theorems in Ch 5.6.

3b) [20 pts - or 25 pts for a correct proof]. See the text. For this question, you could describe β as the number of maximal chains including some element of the given anti-chain (though the textbook proof describes it in a more complex way).