MAD 4302 Oct 24, 2013
Quiz 4 and Key Prof. S. Hudson

1) Show by example that the union of two equivalence relations does not have to be an equivalence relation.

2) Answer True or False to each; you do not have to explain. Let $S = \{1, 2, 3, 4, 5\}$.

$P(S)$ contains a unique antichain of size 10.

$P(S)$ contains an antichain of size 8.

$P(S)$ contains a chain of length 7.

$P(S)$ can be partitioned into 9 chains.

The formula $A^+ \cup A^- = X$ is used in the proof of Sperner's Thm.

3) Determine the number of permutations of $\{1, 2, 3, 4, 5\}$ in which at least one odd integer is in it natural position. Simplify completely.

4) Choose ONE: answer on the back.

Prove the Binomial Theorem using induction, as done in class or in the textbook.

Find / simplify the generating function for the sequence $C(10, k)$, $0 \le k \le 10$.

State the Inclusion-Exclusion Principle. Prove it in the case of only three sets. *Explain your reasoning carefully*; not much credit for just a picture and a few formulas.

Remarks: Each problem counted 25 points. The average on the quiz was 70, with highs of 91 and 87, about the same as Quizzes 2 and 3. The rough scale is the same:

A's 78 - 100
B's 68 - 77
C's 58 - 67
D's 48 - 57

Average grade: I averaged your 4 quiz scores and estimated your current semester letter grade (not yet including HW or extra credit) in the upper left corner of your quiz. You should check this. The average of these is 74, the highest is 86 and the scale is:

A's 81 - 100
B's 72 - 80
C's 62 - 75
D's 52 - 65

1) Let $X = \{1, 2, 3\}$ (Almost any other set is OK. Maybe I should have chosen X for you, to avoid some issues when R and S are not defined on the same X, but I wanted to give you some flexibility). Let $R = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3)\}$ and $S =$

$\{(1,1),(1,3),(3,1),(3,3),(2,2)\}$. Then $(2,1)$ and $(1,3)$ are in $R \cup S$ but $(2,3)$ is not, so $R \cup S$ is not transitive.

A very common mistake was to omit $(3,3)$ from R, but it is required to make R reflexive. I did not deduct much for this. It is a little faster and easier to define R and S by partitions. For example, we can define R quickly by listing its eq. classes; $\{1,2\}$ and $\{3\}$. Some people used very different examples, such as congruences on the integers. That is OK in principle, but it requires at least as much explanation as above, and that was often missing.

2) FTFFF

3) 56. Use I.E. with A_1 = the permu's with 1 in its natural position, etc; $3 \cdot 4! - 3 \cdot 3! + 2 = 56$.

4) Everyone chose (a) and most did OK. The most common problem was inadequate explanation. This proof needs roughly 5 or 6 comments / sentences, in addition to the formulas. See the text or lecture notes.