MAD 4302 Nov 19, 2013
Quiz 5 Prof. S. Hudson

1) [30pts] Solve this: $h_n = -h_{n-1} + 3h_{n-2} + 5h_{n-3} + 2h_{n-4}$ with $h_0 = 1$, $h_1 = 0$, $h_2 = 1$ and $h_3 = 2$. Hint: the characteristic equation has two distinct roots, both in $\{-1, 1, 2, 3\}$. The larger of the two roots has multiplicity 1. Much partial credit for the general solution (without solving for the c_i).

2) [25pts] A polynomial p has degree 3 with values $p(0) = 3$, $p(1) = 6$, $p(2) = 15$, $p(3) = 36$ and $p(4) = 75$. Use a difference table to find a simplified formula for $p(n)$.

3) [20pts] How many ways can the letters $\{a, b, c, d, e\}$ be placed into 3 identical boxes such that no box is empty ? Simplify your answer to an explicit number (such as 24 or 32, for example). For full credit, use standard formulas from Ch 8, rather than listing them all.

4) [25pts] Choose ONE: answer on the back.

a) State Thm 8.1.1 (about ± 1's and Catalan numbers) and give a rough proof. Include the formula for U_n with justification.

b) State Thm 7.6.1 (about triangulations of polygons) and give a rough proof of the recurrence relation for h_n. State the formula for the solution (for h_n) but do not prove that.

c) Compute and simplify $S^{\#}(4, 4) + B_4 + p_4 + h_4^{(2)}$. This is mainly a test of your memory of Ch 8 notation and formulas - so, do not ask for help with that.

Remarks and Answers: The average was approx 71 with high scores of 85, 84, and 83. Your semester average is again in the upper right; the highest of these are 83 and 82, with an average of approx 72. The unofficial scale for the quiz (and the semester) is:

A's 78 - 100
B's 68 - 77
C's 58 - 67
D's 48 - 57

1) I gave approx 25 points for

$$c_1(-1)^n + c_2 n(-1)^n + c_3 n^2(-1)^n + c_4 2^n$$

See the worked out example in Ch 7 for more details and the values of c_i.

2) The 0th diagonal is 3,3,6,6,0 so $p(n) = 3C(n, 0) + 3C(n, 1) + 6C(n, 2) + 6C(n, 3)$ which simplifies to

$$p(n) = n^3 + 2n + 3$$

3) $S(5, 3) = 25$ from recursion formulas or the table (similar to Pascal's Triangle).

4ab) see the text or lectures.

4c) 24+15+5+11 = 55