Study Advice for Ch. 6, MAP 2302

Ch.6 is about series solutions to DE's, mostly 2nd order linear homogeneous ones. You will need to review your series skills from Calc 2 and to study at least 4 examples in some detail to pick up the new skills you may need.

Review problems:

1) Convert $\sum_{k=1}^{\infty} k^{-2}x^k$ to a series of the form $x^c \sum_{j=3}^{\infty} b_j x^j$.

soln: Let $j = k + 2$ and get $\sum_{j=3}^{\infty}(j-2)^{-2}x^{j-2} = x^{-2}\sum_{j=3}^{\infty}(j-2)^{-2}x^j$.

2) Split $\sum_{k=1}^{\infty} k^{-2}$ into 5/4 plus a series of the form $\sum_{j=3}^{\infty} b_j$.

soln: Since $a_1 + a_2 = 5/4$ the new series is the same as the old one, but starting at 3. So $5/4 + \sum_{j=3}^{\infty} j^{-2}$.

3) Find the first 3 nonzero terms of $(1 + 3x + 5x^2 + \ldots)(1 + 2x + 4x^2 + \ldots)$.

soln: Basically, ignore the dots and multiply the two polynomials. But you can omit any large powers of x. So, $1 + 5x + 15x^2 + \ldots$.

4) Find the first 2 nonzero terms of $\frac{1+3x+5x^2+\ldots}{1+2x+4x^2+\ldots}$.

soln: Similar to above but divide poly's. I think $1 + 2x$ into $1 + 3x$ is good enough for 2 terms, which leads to $1 + x + \ldots$. You should be able to get 3 or more terms as well, but this gets tedious.

Summary of Methods: I will not attempt to list all the options and possibilities here, aiming to keep it as simple as possible.

1) At an *ordinary point*, follow the examples of Ch.6.1. Assuming $x_0 = 0$, set $y = \sum c_k x^k$, use the DE to get info about the c_k (this usually leads to a recursion formula). It is usually possible to solve for all the coefficients you want in terms of c_0 and c_1, and those remain in the final answer. Or if IC's are given, you can compute those too. Ex.6.5 is typical.

2) At a *regular singular point* you start with $y = \sum c_k x^{k+r}$ instead (this allows for more variety of solutions). When you set $k = 0$ you should get an equation in r alone, the indicial equation, with two roots, $r_1 \geq r_2$. We will not worry about the case of complex roots (nor about *irregular singular points*). Now there are some sub-cases:

2a) If $r_1 - r_2$ is NOT an integer, we can expect to get a series for y_1 from r_1 and another LI solution y_2 from r_2. Example 6.11 that we did in class is typical. Note that these y_j may not be true power series, because of fractional powers (etc) but they are pretty close to that form.

2b) If $r_1 - r_2$ IS an integer, then we compute y_1 from r_1 as above. I suggest computing y_2 using Reduction of Order. Exercise 25, done in class, is typical (see also Ex.6.14). Expect y_2 to have two parts, the second part including $\ln(x)$. [The author discusses another option, of working with r_2 first, hoping for a shortcut, but I do not recommend this].

2c) The case $r_1 - r_2 = 0$ actually fits into Case 2b) for our purposes. You can use the same method. The expected form for the answer is a little different - compare equations (6.67) and (6.68) in the text, noting that (6.68) has no r_2 and no C. We will see an example of this in Ch.6.3.

Main Examples to study: You can get most of the ideas from the ones done in class, namely Ex.6.5, Ex.6.11 and exercise 6.2.25. We will probably also do an example from Ch.6.3, though that section focuses more on some special solutions (Bessel functions) than on any new solution methods.

I would also strongly suggest reading through the other Examples in the text, though the author is partially trying to show us how things work - how some methods might work better or worse in certain cases. Rough guide to Ch. 6.2:

Ex 6.11 is basic. Know it well.

Ex 6.12 is fairly similar to Ex.6.11.

Ex 6.13 is fairly similar through page 262, but the rest is based on a rather lucky guess (that $C = 0$). I'd use Reduction of Order for Ex. 6.13 instead (you might try that for practice).

Ex 6.14 is similar to ex.6.25, and illustrates Reduction of Order, discussed above. Know.

Now you should be ready for 6.2 HW. And Ch.6.3 does not introduce any new cases or major new methods. It focuses on a very famous DE of the Ch 6.2 type, and its solutions. These *Bessel functions* are roughly speaking the multivariable analogs of $\cos(x)$, etc. The exercises point out some formulas a bit similar to trig identities.