

Correction to an example from class today

Example (similar to Ch.9.1, ex.5) Compute the Laplace transform of

$$f(t) = \begin{cases} t & \text{if } 0 < t < 3 \\ 4 & \text{if } t \geq 3 \end{cases}$$

Solution:  $L(f) = \int_0^3 e^{-st} t \, dt + \int_3^\infty e^{-st} 4 \, dt = A + B$ , where we compute  $A$  using I.B.P.:

$$A = \left. \frac{e^{-st} t}{-s} \right|_0^3 + \int_0^3 \frac{e^{-st}}{s} \, dt = \frac{-3e^{-3s}}{s} - \frac{e^{-st}}{s^2} \Big|_0^3 = \frac{-3e^{-3s}}{s} - \frac{e^{-3s}-1}{s^2} \text{ and}$$

$$B = 4 \left. \frac{e^{-st}}{-s} \right|_3^\infty = 4 \frac{e^{-3s}}{s} \text{ for } s > 0 \text{ (taking a naughty shortcut with the improper integral).}$$

So, the answer is  $\frac{e^{-3s}}{s} + \frac{1-e^{-3s}}{s^2}$ , for  $s > 0$ .

In class, I had  $s$  instead of  $s^2$  in the denominator, probably from some earlier mistake, or miscopying.