MAP 2302 — Sept 25, 2002
Exam I — Prof. S. Hudson

Name

Show all your work and reasoning for maximum credit. If you continue your work on another page, be sure to leave a note. Do not use a calculator, book, or any personal paper. You may ask about any ambiguous questions or for extra paper. If you use extra paper, hand it in with your exam. The problems are 15 points each except that 6) is 10.

1) Find the general solution to this exact DE: $(2x\cos y + 3x^2 y)dx + (x^3 - x^2 \sin y - y)dy = 0$

2) Solve this IV problem: $(y+2)dx + y(x+4)dy = 0$ such that y(-3) = -1.

3) Find the general solution to this equation: $\frac{dx}{dt} + \frac{x}{t^2} = \frac{1}{t^2}$

4) Reduce this equation to one in a simpler form. Mention what the new form is (exact? separable? etc), but you don't have to solve it: $(x - 2y + 1)dx + (4x - 3y - 6)dy = 0$.

5) Find the orthogonal trajectories to this family of curves. You do not have to sketch them: $y^2 = cx$.

6) (10 points) Can we apply the basic existence and uniqueness theorem (Thm 1.1) to the following IV problem ? Explain what (if anything) we can conclude, and why (or why not): $\frac{dy}{dx} = \frac{y}{\sqrt{x}}$ and $y(0) = 2$.

7) Answer True or False - you don't have to explain your answers:

$\frac{dy}{dx} + x^2 y = xe^x$ is a linear first-order ordinary DE.

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ is a linear first-order partial DE.

$\frac{dy}{dx} + y = xy^3$ is a nonlinear Bernoulli DE.

$(5x + y)dx + (2x + 5)dy = 0$ is neither linear nor homogeneous, but has linear coefficients.

Setting $y = vx$ transforms a homogeneous DE ($Mdx + Ndy = 0$)into a separable DE.

Answers:

1) $x^2 \cos y + x^3 y - y^2/2 = c$. This is Ex 2.6 on page 32 (without the initial conditions).

2) $x + 4 = (y+2)^2 e^{-(y+1)}$. This is exercise 15 on page 47. One key step, after separating the variables, is $\int \frac{y}{y+2}\, dy = \int \frac{y+2-2}{y+2}\, dy = \int 1 - \frac{2}{y+2}\, dy = y - 2\ln|y+2| + C$. (You could also integrate this using the substitution $u = y + 2$).

3) $x = 1 + ce^{1/t}$. This is exercise 5 on page 56. You can solve it as a linear DE (using Thm 2.4). But it's probably easier to separate variables: $\frac{dx}{1-x} = \frac{dt}{t^2}$ etc. Of course, $\int t^{-2}dt = -t^{-1} + C$.

4) This is Example 2.19 on page 65. You can stop when you get to equation 2.54, $(X - 2Y)dX + (4X - 3Y)dY = 0$, and say it is homogeneous.

5) This is exercise 2 on page 79. Remember that you have to treat c with more respect in these problems than in Ch.2. Apply $\frac{d}{dx}$ to the family and get $2y\ dy/dx = c = y^2/x$ or $dy/dx = y/2x$. For the O.T's, we get $dy/dx = -2x/y$ or $y\ dy = -2x\ dx$. Integration now gives the answer, $y^2 + 2x^2 = c$.

6) We can't make any conclusion, because f is not continuous at 0. This is Example 1.16, part 2, on page 21.

7) TFTFT (due to a grading mistake, TFTTT was also marked OK).