MAP 2302 — Sept 18, 2014
Exam 1 and Key — Prof. S. Hudson

Typo's have been corrected in problem 1) and the bonus.

1) [30pts] For each DE below, state what type it is (homogeneous, Bernoulli, 'Ch. 2.4A', or etc). Then, find an integrating factor $\mu(x, y)$ to make the DE exact. You do not have to solve the DE. As always, show your work and/or explain your reasoning.

1a) $\frac{dy}{dx} + 3x^2y = 5x$

1b) $x(y + 1)\ dx + (xy + y)\ dy = 0$

1c) $[y^2(x + 1) + y]\ dx + [2xy + 1]\ dy = 0$. Hint, this has a μ that depends only on x.

2) [15pts] Solve the initial value problem: $2xy\ dx + (x^2 + 1)\ dy = 0$ with $y(1) = 3$.

3) [10pts] Solve the initial value problem: $y'' - y' - 12y = 0$ with $y(0) = 6$ and $y'(0) = 10$. HINT: the general solution to the DE is $c_1e^{4x} + c_2e^{-3x}$.

4) [10pts] A stone weighing 4 lbs falls from rest toward the earth from a great height. As it falls it is acted on by air resistance numerically equal to $v/2$ (in lbs), where v is the velocity (in feet per second). Set this up as an IVP in v and t that would allow you to solve for v (but you do not have to solve it). You can use standard formulas such as $F = ma$ and $g = 32$.

5) [15pts] Determine the value of the constant A such that $[Ax^2y+2y^2]\ dx+[x^3+4xy]\ dy = 0$ is exact. Solve the resulting DE.

6) [20pts] Answer each with True or False in the margin. You do not have to explain, unless the problem is ambiguous. Interpret *the best way* to mean *a standard efficient way.*

The IVP $\frac{dy}{dx} = x^2 + y^2$ with $y(1) = 3$ has a unique solution, defined on some open interval of the form $(1 - h, 1 + h)$.

The equation $y'' + xyy' + 4y = 2x$ is a linear 2nd-order ordinary differential equation.

The equation $\frac{\partial v}{\partial s} + \frac{\partial v}{\partial t} = v$ is a partial differential equation.

The best way to approach the DE $\frac{dy}{dx} + y = xy^3$ is to set $y = vx$, to make it linear.

The best way to approach the DE $(x+2y+3)\ dx+(2x+4y-1)\ dy = 0$ is to set $z = x+2y$, to make it separable.

Bonus [5pts, maybe hard]: Find the general solution of $y'' - 7y' + 10y = 0$, given that there are two solutions of the form $y = e^{kx}$.

Remarks, Scale and Answers: The average among the top half of the class was approx 84, which is unexpectedly good, with high scores of 103 and 98. The average score on every problem was fairly good, the lowest being 68% on #4 (and 35% on the bonus, which I actually consider to be good). Unfortunately, the (unofficial) scale for this exam must be a bit higher than expected too. The overall scale at the end will probably be closer to the one on the syllabus. For this exam:

A's 92-100
B's 82-90
C's 72-81
D's 62-71

Many people did not read the instructions. If you answer a question anywhere except the space provided, you must leave a note in the space below the problem, about where to look. Else, I probably won't see your answer and you will get a zero. Also, many people solved DE's when they weren't asked to do so (or vice-versa). Some people did reasonably good work on a problem, but inexplicably did not answer the question in the proper form.

1a) Linear. $\mu = e^{x^3}$. Most people got this.

1b) This is easy! Or, it was supposed to be. It is separable, so we multiply by $\mu = \frac{1}{(x+1)(y+1)}$ to separate the x's and y's. Many people multiplied by this correctly (and even solved the DE), but did not seem to realize they were using an integrating factor, and never really answered the question. If you DID realize this, you should have labeled your μ clearly for full credit.

1c) It is the 2.4.A type, with $\mu = e^x$. I also accepted labels such as "Special Integrating Factors", but did not accept "the integrating factor depends only on x" since that phrase is given. If you did not say anything about the type, you lost about 4 points. Make sure you answer all parts of every question !

2) $(x^2 + 1)y = 6$. The DE is separable and fairly easy. The most common errors were algebraic (rules for exponents and logs).

3) $y(x) = 4e^{4x} + 2e^{-3x}$. You can actually ignore the DE and just work with the IC's to get $c_1 = 4$ and $c_2 = 2$. But your final answer should have the form $y(x) = \ldots$.

4) From $ma = mg - v/2$, get $v'(t) = 32 - 4v(t)$ with the IC, $v(0) = 0$. The most common errors were algebra, or $+4v(t)$ or forgetting the IC. Many people solved the IVP, but that was not required (and not graded).

5) $A = 3$ (5 pts) and $x^3y + 2y^2x = C$ (10 pts), from the usual $F = \int M$ method. Including 'F' in the answer is optional. Common mistakes were to write $x^3y + 2y^2x + C$ (minor) or neglecting to solve the DE at all (major).

6) TFTFT

B) $y(x) = c_1e^{2x} + c_2e^{5x}$. Many people got to $y = e^{2x}$ and $y = e^{5x}$ for partial credit, but this is called a fundamental set of solutions, and is not (quite) the same as the general solution.