MAP 2302 Oct 21, 2002
Exam II + Key Prof. S. Hudson

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1) Given that $y = x$ is a solution of $x^2y'' - 4xy' + 4y = 0$, find the general solution by reducing the order.

2) Use the method of UC's to find a particular solution of the DE: $y''+2y'+2y = 10\sin(4x)$. Small help 1), $y_c = e^{-x}(c_1\sin(x) + c_2\cos(x))$. Small help 2), You can stop when you get to the two equations in two unknowns (”A” and ”B”, no x's).

3) Choose ONE proof. Explain thoroughly. All three refer to the usual DE: $a_0(x)y'' + a_1(x)y' + a_2(x)y = 0$ on an interval [a,b].

a) Prove 4.18A: if f_1 and f_2 are solutions of the DE, and $W(x) = 0$ for all x in [a,b], then f_1 and f_2 are LD.

b) Prove 4.18B: if f_1 and f_2 are LD solutions of the DE, then $W(x) = 0$ for all x in [a,b].

c) Prove 4.6.20: if the DE has two LI solutions then every other solution is a LC of those two.

4) Find the general solution of $2y'' + y' - 6y = 0$.

5) Solve the DE: $y'' - 2y' + y = xe^x\ln(x)$ $(x > 0)$. Small help 1), the characteristic equation is $m^2 - 2m + 1 = (m-1)^2 = 0$, which has a double root. Small help 2), stop once you have a formula for $v_1'(x)$.

6) Solve this Cauchy-Euler I.V.P for $x > 0$: $x^2y'' + 5xy' + 3y = 0$ with $y(1) = 1$ and $y'(1) = -5$.

The average was about 80/100 or more on all problems except 3). Answers (edited 7/17/13):

1) (pg 132 #1) Use $y = vf = vx$ and $v' = w$ to get $x^3w' - 2x^2w = 0$. Then $v = \int x^2 = cx^3$, so $y = c_1x + c_2x^4$.

2) (pg 160 #4) Set $y = A\sin 4x + B\cos 4x$, find y' and y'' and put into the DE to get $-14A - 8B = 10$ and $8A - 14B = 0$.

3) (these are theorems from ch 4.6) I suggest trying very hard to *understand* the assigned proofs before memorizing them. In fact, you may have to memorize very little once the main ideas are clear. For example, in thm 4.18A, the main ideas are to quote thm A, thm 4.15 and thm B (in that order!). You do not have to know the theorem numbers and can write, for example, ”by a theorem of algebra” in your proof.

4) (pg 143, #5) $y = c_1 e^{3x/2} + c_2 e^{-2x}$

5) (pg 170, #10) Start with $y_1 = e^x$ and $y_2 = xe^x$ and get $y_2' = e^x + xe^x$. Using the standard formula (4.62) on page 164 (or the 'full method' used in Ex.4.40), $v_1'(x) = F(x)y_2(x)/W(x) = -x^2 \ln(x)$.

6) (pg 177 #25, HW, and done in class) Set $x = e^t$ and get $y'' + 4y' + 3y = 0$. Then $y = c_1 e^{-3t} + c_2 e^{-t} = c_1 x^{-3} + c_2 x^{-1} = 2x^{-3} - x^{-1}$.