MAP 2302 Oct 19, 2015

Exam 2 Prof. S. Hudson

1) [15 pts] Find the general solution to $y^{iv} - y = 0$ using Ch.4.2 methods. Hint: if you get stuck on the algebra, the formula for $a^2 - b^2$ may help. It can be used on $x^4 - y^4$, etc.

2) [15 pts] Find the general solution using the UC method of Ch 4.3. Hint $y_c = c_1 e^{-x} + c_2 e^{-5x}$.

$$y'' + 6y' + 5y = 2e^x + 10e^{5x}$$

3) [20 pts] A 12-lb weight is placed on the lower end of a coil spring suspended from the ceiling. The weight comes to rest in its equilibrium position, thereby stretching the spring 1.5 in. The weight is then pulled down another 2 in, and released from rest at time $t = 0$. Find the displacement (in feet) as a function of t, and the amplitude of the motion. Assume no air resistance or other force is present. Assume $g = 32$ ft/sec/sec.

4) [10 pts] In each part, classify x_0 as an ordinary point, a regular singular point, or an irregular singular point.

a) $x^2y'' + (x+2)y' + (x+3)y = 0$, $x_0 = 0$.

b) $(x^2+1)y'' + (x-1)y' + (x+3)y = 0$, $x_0 = 1$.

5) [25 pts] Answer each with True or False. You do not have to explain.

In free damped motion, $x(t) = 0$ can occur at most once.

If f_1, f_2 and f_3 all solve $(x^2+1)y'' + (x-1)y' + (x+3)y = 0$, then f_3 is a linear combination of f_1 and f_2.

If f_1, f_2 and f_3 all solve $(x^2+1)y'' + (x-1)y' + (x+3)y = 0$, then $\{f_1, f_2, f_3\}$ is LD.

Setting $x = e^t$ reduces $xy' + 2y = \sin(x)$ to a linear DE with constant coefficients.

Variation of parameters can be applied to $y''' - 6y'' + 11y' - 6y = e^x$.

6) [15 pts] Choose ONE proof. These were both done in class (Thms FIU 1 and FIU 2) and are essentially the 2 halves of Thm 4.18. They refer to two solutions f_1 and f_2 of the usual Ch.4.6 DE, $a_0(x)y'' + a_1(x))y' + a_2(x)y = 0$.

A) If $W(x_0) = 0$ for some point x_0 in the domain, then f_1 and f_2 are L.D.

B) If f_1 and f_2 are L.D., then $W(x) = 0$ for every point x in the domain,

Bonus) [5 pts] Find the general solution of $(x^2+1)y'' - 2xy' + 2y = 0$ using Ch 4 methods.

Remarks and Answers: The top 22 average was approx 66 / 100 with high scores of 89 and 81, which is a fairly normal result and similar to Exam I. The scores were good on page 1 (about 85%), moderate on page 2 (about 65% to 70%), and pretty bad on the proof (about 35%). The bonus was intended to be harder, but approx 10 people got some points for finding $y = x$. The advisory scale for Exam 2 (and for your Exam 1-2 average so far) is:

A's 75 to 100
B's 65 to 74
C's 55 to 64
D's 45 to 54

1) $y(x) = c_1e^{-x} + c_2e^x + c_3 \sin(x) + c_4 \cos(x)$. The char eqn is $m^4 - 1 = 0$, which factors into $(m - 1)(m + 1)(m^2 + 1) = 0$ with four roots ± 1 and $\pm i$. Common mistakes were $m^4 - m = 0$ and failure to look for FOUR roots.

2) $y(x) = c_1e^{-x} + c_2e^{-5x} + \frac{1}{6}e^x + \frac{1}{6}e^{5x}$.

3) This was an assigned HW problem, 5.2.1. $x(t) = \frac{1}{6}\cos(16t)$ and the amplitude is $\frac{1}{6}$. Some steps are $k = 96$, $m = 3/8$, $x'' + 2^8x = 0$, $x(t) = c_1 \cos(16t) + c_2 \sin(16t)$, $x(0) = 1/6$, $x'(0) = 0$. Remember to convert inches to feet (since those units are used in g, and the problem insists on feet).

4a) irreg. singular, 4b) ordinary. No partial credit. I sometimes insist on justifications, but did not this time.

5) FFTTT

6) See the lecture notes or the text. The results were not very good, as if people did not study these much, and/or had no feel at all for proofs. There were quite a few answers, not labeled A or B, that did not seem to match either. Many answers included no words. When writing a proof, you must always try to *communicate*.

To prepare for a proof, I'd suggest first *understanding* it, perhaps by getting help. Then decide how much of it you can reproduce easily (even a few days later, under pressure). Then, you may have to memorize a few key steps, but try to keep this to a minimum. When you think you have it, practice writing it out without notes. Check that your proof has enough words, from top to bottom, for the average reader to follow your thinking.

If I give you a list of 4-5 proofs for an exam, you may not have the time or ability to learn them all 100%. It is probably better to learn half the list well than the whole list badly.

Bonus) Guess that $y = x$ is a solution and then reduce the order. See Ex.4.16.