MAP 2302 Mar 3, 2016
Exam 2 and Key Prof. S. Hudson

1) [10 pts] A stone weighing 4 lbs falls from rest towards the earth from a great height. Assume air resistance is equal to $\frac{v}{2}$ with v = velocity in ft/sec. Solve a DE to find a formula for $v(t)$.

2) [15 pts] Answer True or False. You do not have to explain, unless you think some part is ambiguous.

If y_1 and y_2 are particular solutions of a 3rd order linear homogeneous DE with variable coefficients, then $y_1 - y_2$ is also a solution.

The function $F(x) = xe^x \sin(2x)$ is a UC function.

The UC set of x^2e^x contains exactly 4 functions.

A free undamped motion could satisfy $x'' + x' + 2x = 0$.

If a free damped motion satisfies $x''+8x'+9x = 0$, then it is under-damped and oscillatory.

3) [10 pt] A 12-lb weight is attached to the lower end of a coil spring suspended from the ceiling. It comes to rest after stretching the spring 1.5 in. It is then pulled down another 2 in and released from rest at $t = 0$.

Set this up as a free undamped motion IVP as in Ch.5.2. You can ignore air resistance and do not have to solve the DE or the IVP. But include the usual constants such as k, m, w and $g = 32$ in your work.

4) [15 pts] Solve the IVP $y'' - 4y' + 29y = 0$, $y(0) = 0$, $y'(0) = 5$.

5) [15 pts] Given that $y_1(x) = x$ is a solution of $(x^2 - 1)y'' - 2xy' + 2y = 0$, find the general solution using reduction of order. You can use the usual shortcut. If you do not, then [small hint] the formula $\frac{-2}{x} + \frac{1}{x+1} + \frac{1}{x-1}$ may arise, about halfway through.

6) [10 pts] Find the general solution of the Cauchy-Euler DE, $x^3y''' - 4x^2y'' + 8xy' - 8y = 4$ for $x > 0$. You may use the shortcut mentioned in class (which is based on expanding $r(r - 1)$ and/or $r(r - 1)(r - 2)$). Hint: A char. eqn. should arise with 3 distinct roots among these 4 numbers $\{1, 2, 4, 8\}$ (one of the 4 is wrong). After the shortcut, show all your work, not relying *entirely* on the hint.

7) [10 pts] Choose one proof. These refer to the usual DE from 4.6, $a_0(x)y'' + a_1(x)y' + a_2(x)y = 0$, with continuous coefficients, with $x > 0$ and $a_0(x) > 0$. As usual, LI and LD abbreviate *linearly (in)dependent*.

a) Prove Thm 4.18A, that if $W(x_0) = 0$ then the solutions f_1 and f_2 are L.D.

b) Prove Thm 4.16, that the DE has a LI pair of solutions f_1 and f_2.

8) [15 pts] Find the general solution of $y'' + y = \tan(x)$ by variation of parameters. Do it the long way, using these formulas; $y_c(x) = c_1 \sin(x) + c_2 \cos(x)$ and the imposed condition $v_1'(x)\sin(x) + v_2'(x)\cos(x) = 0$. If you are short on time, you can use the shortcut instead (the integrals for the v_j) but for a maximum of 10 points.

Bonus [5pts]: Transform $(x + 2y + 3)dx + (2x + 4y - 1)dy = 0$ into a separable DE. Write out the new DE in a standard form, but do not solve it.

Remarks and Answers: The average was 70 out of 100 with high scores of 98 and 93, which is fairly good, clearly better than Exam 1. The average on each problem varied from 52% on # 6 to 85% on # 4. Most of the problems came from textbook exercises or Examples, such as these; 3.2.1, Ex 4.40, 4.1D.3, 4.2.45, Ex 4.44 simplified, 5.2.1 simplified. Here is an unofficial scale for the exam.

A's 78 to 100
B's 68 to 77
C's 58 to 67
D's 48 to 57

For a semester estimate so far, you can average your Exam 1 and 2 scores and use this scale:

A's 76 - 100
B's 65 - 75
C's 55 - 64
D's 45 - 54

1) The DE is $\frac{dv}{dt} = 32 - 4v$, with $v(0) = 0$, and the solution is $v(t) = 8 - 8e^{-4t}$.

2) TTFFF

3) $x'' + 256x = 0$, $x(0) = 1/6$, $x'(0) = 0$. This uses $w = 12$, $m = 3/8$, $k = 96$ and $mg = kl$ (etc).

4) $y(x) = e^{2x}\sin(5x)$. Get $m = 2 \pm 5i$ so $y(x) = e^{2x}[c_1 \sin(5x) + c_2 \cos(5x)]$. Then use the IC's.

5) $y(x) = c_1 x + c_2(x^2 + 1)$. Most people who did not use the shortcut made small errors along the way (such as $\int x^{-2}\,dx = x^{-1}$) but got partial credit. Either way, you should get $w = 1 - x^{-2}$ near the end. Another common understandable mistake was forgetting to write out the *general* solution.

6) $y(x) = c_1 x + c_2 x^2 + c_3 x^4 - 1/2$. As usual, set $x = e^t$ and convert to a CC DE in y, t (probably using the shortcut). The char. eqn. is $m^3 - 7m^2 + 14m - 8 = 0$ with roots 1, 2 and 4. After changing e^t back to x, we get $y_c(x) = c_1 x + c_2 x^2 + c_3 x^4$. Then use the UC method (is it very quick for this one) to get $y_p(x) = A = -1/2$. Many people seemed unprepared for this kind of example, but some also stopped too early with answers like $y_c(x) = c_1 e^x + \cdots$. We did almost this same example in class.

7) See my lectures. Or, the text ('Thm 4.18A' is roughly the first half of Thm 4.18, plus a little more thought).

8) $y = c_1 \sin x + c_2 \cos x - \cos x \ln|\sec x + \tan x|$. This is Ex.4.40, which we did in class (but probably using the shortcut).

B) This was on Exam I.