MAP 2302 — Nov 18, 2002
Exam III + Key — Prof. S. Hudson

1a) (5 pt) Identify the singular points of the DE and state whether the method of Frobenius can be applied at those points:

$$(x^3 + x^2)y'' + (x^2 - 2x)y' + 4y = 0$$

1b) (5 pt) Find and classify the singular point(s) of the DE: $x^2y'' + x(x-1)y' - 8y = 0.$

1c) (10 pt) Find the indicial equation of the DE of 1b) at $x_0 = 0$ and its roots.

2) (10 pt)Choose ONE and provide the correct formula.

a) $J_0(x) = ?$ (give the power series at 0).

b) $\Gamma(N) = ?$ (give the integral definition).

3) (15pt) Choose ONE proof. Explain thoroughly. Both refer to the DE: $a_0(x)y'' + a_1(x)y' + a_2(x)y = 0$ on an interval [a,b].

a) 4.6.16: If the coefficients are continuous and nonzero, the DE has two LI solutions on [a,b].

b) 4.6.20: if the DE has two LI solutions then every other solution is a LC of those two.

4) (20pt) Answer with True or False.

If y_1 and y_2 are any two LI solutions of Bessel's equation of order p (where $p > 0$ is any real number), then at least one of them contains a logarithm term.

The positive zeros of J_0 and Y_0 separate each other.

The Bessel DE of order 1/2 has two LI solutions in closed form (not a series).

If the indicial equation is $r^2 - 3r - 4$, we won't need reduction of order.

At an ordinary point x_0 of the usual DE (see 3)), there is a power series solution that converges for some $x \neq x_0$.

5) (15pt) A 16 lb weight is attached to the lower end of a coil spring. In equilibrium, the spring is stretched 6 inches. The weight is then pulled down 3 inches below that position and released at $t = 0$. The medium offers a resistance in pounds equal to $10x'$ where x' is the velocity in feet per second.

Set up the appropriate initial value problem, but do not solve.

6) (20pt) Find a power series solution of this IVP [write out the first three nonzero terms]:

$$y'' + xy' - 2y = 0, \quad y(0) = 0, \quad y'(0) = 1$$

Answers: 1a) The singular points are at 0 and -1, both are regular, so the method can be used.

1b) $x = 0$ is a regular singular point.

1c) $r^2 - 2r - 8 = 0$ has roots $r = 4$ and $r = -2$.

2) and 3) See text

4) FTTFT. Since the grades were low on this one, here are some explanations:

a) If $p = 1/3$, for example, then $r_1 - r_2$ is not an integer, and there will be no log.

b) See the diagram or the reading in the text.

c) See HW 6.3.3 and the answer on page 578 (which involves trig and a square root, but no series).

d) See case 2 of thm 6.3, page 257, or examples like 6.14 on page 265. We *might* need R of O.

e) This is the main theorem of Ch 6.1.

5) [see 5.3-4] $\frac{1}{2}x'' + 10x' + 32x = 0$ with $x(0) = 1/4ft$ and $x'(0) = 0$.

6) [see 6.1-16] The recurrence relation is $c_{n+2} = -\frac{(n+2)(n+1)c_n}{(n-2)}$. We get $c_0 = 0$ and $c_1 = 1$ and $y = x + x^3/6 - x^5/120 + \ldots$