MAP 2302 Nov 16, 2015
Exam 3 Prof. S. Hudson

1) Rewrite (99.5)(98.5)(97.5)(96.5)(95.5) using Gamma function(s).

2) Find and solve the indicial equation for this DE, at the singular point $x_0 = 0$. You do not have to solve the DE:

$$3xy'' - (x-2)xy' - 2y = 0$$

3) Find two LI series solutions to Ex.6.5 from Ch 6.1, $y'' + xy' + (x^2+2)y = 0$. To save time, the main formulas from the first phase are

$2c_0 + 2c_2 = 0$, $\quad 3c_1 + 6c_3 = 0$, $\quad$ and

$(n+2)(n+1)c_{n+2} + (n+2)c_n + c_{n-2} = 0$, for $n \geq 2$.

4) Find the first two nonzero terms of the McLaurin series (the series centered at 0) for this function

$$f(x) = \frac{2x + 4x^2 + 6x^3 + \ldots}{1 + 3x + 5x^2 + \ldots}$$

5) Compute the Laplace transform $L\{e^{2t}\sin(t)\}$. You may use a memorized formula from Table 9.1 to save time, but if so, state the formula that you are using (this also applies to problems below).

6) Compute the inverse Laplace transform $L^{-1}\{\frac{1}{(s+2)(s+3)}\}$ using partial fractions.

7) Compute the convolution $h(t) = f * g(t)$, where $f(t) = e^{5t}$ and $g(t) = e^{2t}$. Then find the Laplace transform $L\{h(t)\}$.

8) Start solving this IVP using the Laplace transform, but stop when you have a formula for $Y(s)$. You do not have to use partial fractions or a convolution, etc.

$y'' - 2y' - 8y = 0$, $y(0) = 3$, $y'(0) = 6$.

9) [20 pts] Answer each with True or False.

At an ordinary point x_0 of a standard Ch.6 DE, there is a power series solution that converges for some $x \neq x_0$.

We can get the series for $J_{-p}(x)$ by substituting $-p$ for p in the series for $J_p(x)$.

For all $x > 0$, $[J_0(x)]^2 + [Y_0(x)]^2 = 1$

$f(t) = e^{4t}\sin(t) + 2^t$ has exponential order.

If f is a positive bounded continuous function, then L(f) is also positive.

Bonus) [5 pts] State (or derive) line 13 of Table 9.1, related to $\frac{1}{(s^2+b^2)^2}$.

Remarks and Answers: The average among the top half was 60/100, with highs of 85 and 73. The results were poor on problems 1 and 2, stronger on problem 6. The scale for the exam is:

A's 70 - 100
B's 60 - 69
C's 50 - 59
D's 40 - 49

The exam average for the semester is approx 63 with this scale:

A's 71 - 100
B's 61 - 70
C's 51 - 60
D's 41 - 50

1) $\frac{\Gamma(100.5)}{\Gamma(95.5)}$. This is similar to the formula $7 \cdot 6 \cdot 5 = \frac{7!}{4!}$. Another possible answer is $\frac{99.5!}{94.5!}$, except for the instructions to use Gamma functions.

2) $3r(r-1)$ with roots $1, 0$ (Solution corrected on 4/11/16. The question was supposed to match ex.6.2.11). There seemed to be much confusion about what to do. I gave partial credit for writing down r somewhere, but roughly half did not do that.

3) $c_0(1 - x^2 + x^4/4 + \cdots) + c_1(x - x^3/2 + 3x^5/40 \cdots)$. Use the given info to get $c_2 = -c_0$, $c_4 = c_0/4$, etc.

4) $2x - 2x^2 + \cdots$ Very few people got anywhere on this one. Review how to divide polynomials from your Precalculus or Calculus books (see the chapters on partial fractions and/or power series).

5) Translate the transform of $\sin(t)$ to get $\frac{1}{(s-2)^2+1}$. You can quote the Table instead, if you remember it.

6) P.F's lead to $\frac{1}{s+2} - \frac{1}{s+3}$, so the transform is $e^{-2t} - e^{-3t}$.

7) The convolution is $[e^{5t} - e^{2t}]/3$.

The transform is $[\frac{1}{s-5} - \frac{1}{s-2}]/3$. I slightly prefer using the convolution theorem, which gives $\frac{1}{s-5} \cdot \frac{1}{s-2}$, which is the same thing.

If you could not compute the convolution directly, you could take the inverse transform of $\frac{1}{s-5} \cdot \frac{1}{s-2}$ (using P.F's).

8) $Y(s) = \frac{3s}{s^2-2s-8}$

9) TTFTT