MAP 2302 April 7, 2016
Exam 3 Prof. S. Hudson

1) [15 pts] An 8-lb weight is attached to the lower end of a coil spring suspended from the ceiling. It comes to rest after stretching the spring 6 in. We will call this position $x = 0$. It is then pulled down another 9 in and released from rest at time $t = 0$. The medium offers resistance of $4x'$ pounds (where $x'(t)$ is the velocity in ft/sec). Set this up as a free motion IVP as in Ch.5.3 with the usual variables, x for displacement in feet and t for time in seconds. For maximum credit, work this out from basic principles rather than a memorized formula. You do not have to solve the IVP. Include the usual constants such as k, m, w and $g = 32$ ft/sec/sec in your work.

2) [15 pts] Start working out Ex 6.6, to find a power series solution to this IVP, but you can stop as soon as you have found c_2 and c_3.

$$(x^2 - 1)y'' + 3xy' + xy = 0 \quad \text{with} \quad y(0) = 4 \quad \text{and} \quad y'(0) = 6$$

3) [15 pts] Ex.6.12. Use the Method of Frobenius to find the general solution of $2x^2y'' + xy' + (x^2 - 3)y = 0$ on some interval $0 < x < R$ (so $x_0 = 0$ which is a singular point). To save time, here is some of the early work - you only need to finish it: $r_1 = 3/2$, $r_2 = -1$, $y_1(x) = x^{3/2}(1 - \frac{x^2}{18} + \cdots)$ For $y_2(x)$ you can use reduction of order if you like. Or, you can use $c_1 = 0$ and the recurrence formula, $n(2n - 5)c_n + c_{n-2} = 0$, for $n \geq 2$. Include the first 3 nonzero terms of y_2. Discuss briefly whether y_1 and y_2 are L.I.

4) Short answer [5 pts each]. Be precise. But you can use notation such as $n!$, $b^2 - \lambda^2$, y_1, $F(s)$, *L.I.* and *analytic* without explanation, if you need these.

Define the phrase *under-damped motion* as used in Ch. 5.

Define the phrase *ordinary point* as used in Ch. 6.

State Bessel's DE of order 3.

State the definition of $\Gamma(N)$.

State the formula (in Theorem 9.3) for the Laplace transform of $f'(t)$.

5) [5 pts] Explain what happens when the author tries to solve Ex.6.14 without using reduction of order. Does he find particular solutions $y_1(x)$ and $y_2(x)$ this way ? Does he find the general solution this way ? You probably don't need to recall the Example in any detail, except perhaps that $r_1 - r_2 = 3 - 1 = 2$.

6) [10 pts] Use the definition of the Laplace transform to find $L\{t\}$ at s. State the domain and show all the work.

7) [15 pts] Answer True or False. You do not have to explain, unless you think some part is ambiguous.

$J_0(x)$ is the Bessel function $y_1(x)$ from the method of Frobenius (with $p = 0$, $c_0 = 1$).

$Y_0(x)$ is the Bessel function $y_2(x)$ from the method of Frobenius (with $p = 0$, $c_0 = 1$).

$Y_3(x)$ contains a $\ln(x)$ term.

$\Gamma(4) = 4\Gamma(3)$

$\Gamma(1/2) = 1/2$.

Bonus [5pts]: Prove the formula relating $\Gamma(N + 1)$ and $\Gamma(N)$.

Remarks and Answers: The average was 71% with high scores of 94 and 91, which is fairly good. The results were similar on all problems, but a bit lower on 2 and 4, higher on 1 and 7. The scale for the exam is:

A's 77 to 100
B's 67 to 76
C's 57 to 66
D's 47 to 56

For an estimate of your semester grade so far, average your 3 exam scores and use the scale below. Of course, the final exam, HW, EC, etc will be included later. The best semester averages are currently 88 and 87, with an average average of 65 among the top 20 students.

A's 72 to 100
B's 62 to 71
C's 52 to 61
D's 42 to 51

1) $x''(t) + 16x'(t) + 64x(t) = 0$ with $x(0) = 0$ and $x'(0) = 3/4$. This is based on ex. 5.3.3, which was assigned as HW. For full credit, you were supposed to briefly review the 3 forces acting on the object (see pages 190-191) rather than simply plug into formula (5.7). That part was worth about 3 points.

2) See the text. $c_2 = 0$ and $c_3 = 11/3$. A common mistake was ignoring the ICs, which tell us that $c_0 = 4$ and $c_1 = 6$. A few people did not realize that 0 is an ordinary point and used x^{k+r} as in Ch.6.2, which did not work out well.

3) See Ex.6.12 in the text. You can present your final answer that way, $y(x) = C_1 y_1(x) + C_2 y_2(x)$, along with a formula for y_2 like (6.75). Or, you can write it all out as done in Ex.6.11. The solutions are LI by Thm 6.3, part 1, or you can just observe that they are not scalar multiples of each other.

Most people realized that Reduction of Order is a bad idea here. It should work, but it probably requires some tedious power series manipulations. Nobody got it right that way.

4a) This motion occurs when $b < \lambda$ (or, you can say $b^2 - \lambda^2 < 0$). Normally I would expect more explanation, in words, but the instructions to problem 4 allow this kind of short answer. I gave partial credit for some more wordy explanations (such as 'there is just a little air resistance'), but it is hard to define the phrase precisely that way.

4b) It is an ordinary point if P and Q are analytic at x_0. Many people probably had the right idea but wrote that 'the whole DE is analytic', or 'the point is analytic', etc.

5) Yes, he finds $y_1(x)$ and $y_2(x)$ this way but they are not LI, so they do not provide the general solution. So, he switches to reduction of order to find a better $y_2(x)$, and then succeeds that way.

6) $F(s) = \int_0^\infty e^{-st} t \, dt = \cdots = 1/s^2$ for $s > 0$. Use IBP and show all the work.

7) TFTFF

B) $\Gamma(N+1) = \int_0^\infty e^{-x} x^N \, dx = \cdots = N\Gamma(N)$, using IBP.