MAP 2302 Dec 11, 2002
Final Exam Prof. S. Hudson

Name

Show all your work and reasoning for maximum credit. If you continue your work on another page, be sure to leave a note. Do not use a calculator, book, or any personal paper. You may ask about any ambiguous questions or for extra paper. If you use extra paper, hand it in with your exam.

1A) (10pts) Solve the DE: $(x^2 - 3y^2)\,dx + 2xy\,dy = 0$.

1B) (10pts, Ex. 4.37) Begin the UC method of solving the DE. Write down the correct form of y_p [with an A and a B (etc), but do not solve for A (etc)].

$$y'' - 3y' + 2y = 2x^2 + e^x + 2xe^x + 4e^{3x}$$

1C) (10pts, Ex 4.16) Check that $y = x$ is a solution of $(x^2+1)y'' - 2xy' + 2y = 0$, and find another LI solution by reducing the order.

2) (10pts) Given that $x = 0$ is a regular singular point of the equation

$$xy'' + y' + 2y = 0$$

a) find the roots of the indicial equation.

b) Show the general form of the solutions. [You do not have to solve the DE to do this, if you remember Thm 6.3].

3) (10pts) Choose ONE proof. Explain thoroughly.

a) Thm 9.8: $L\{f * g\} = L\{f\}L\{g\}$.

b) Thm 9.9: $L\{u_a(t)f(t-a)\} = e^{-as}F(s)$

4) (15pt) Answer with True or False.

If f is a positive function, then L(f) is also positive.

If f is a continuous function, then L(f) is also continuous.

If f is periodic then L(f) is periodic.

The inverse LT of Y cannot have two different answers.

At an ordinary point x_0 of the usual DE, there is a power series solution that converges for some $x \neq x_0$.

5) (10pts, ex: 3.2.1) A stone weighing 4 lb falls from rest from a great height. As it falls, it is acted on by air resistance that is numerically equal to $v/2$, where v is velocity. Find the velocity and distance fallen at time t.

Set up the appropriate initial value problem, but do not solve.

6) (10pts, ex: 9.5.1) Solve the system using the Laplace transform:

$$\begin{aligned} x' + y &= 3e^{2t} \\ y' + x &= 0 \\ x(0) = 2, &\quad y(0) = 0 \end{aligned}$$

7) (10pts, Ex: 9.22) Find the inverse Laplace transform of the function below (small hints included, but you'll need to find A):

$$\frac{s^2+5s-3}{(s+2)(s-1)^2} = \frac{A}{s+2} + \frac{B}{s-1} + \frac{C}{(s-1)^2} \text{where} \quad A = ??, \quad B = 2, \quad C = 1$$

8) (5pts, ex: 9.4.3) Find the Laplace transform of this piecewise-defined function: $f(t) = 4$ if $0 < t < 6$ and $f(t) = 0$ if $t > 6$.