Name

Show all your work and reasoning for maximum credit. If you continue your work on another page, be sure to leave a note. Do not use a calculator, book, or any personal paper. You may ask about any ambiguous questions or for extra paper. If you use extra paper, hand it in with your exam.

1A) (10pts) Solve the DE: \((x^2 - 3y^2) \, dx + 2xy \, dy = 0\).

1B) (10pts, Ex. 4.37) Begin the UC method of solving the DE. Write down the correct form of \(y_p\) [with an \(A\) and a \(B\) (etc), but do not solve for \(A\) (etc)].

\[ y'' - 3y' + 2y = 2x^2 + e^x + 2xe^x + 4e^{3x} \]
1C) (10pts, Ex 4.16) Check that \( y = x \) is a solution of \((x^2 + 1)y'' - 2xy' + 2y = 0\), and find another LI solution by reducing the order.

2) (10pts) Given that \( x = 0 \) is a regular singular point of the equation

\[ xy'' + y' + 2y = 0 \]

a) find the roots of the indicial equation.

b) Show the general form of the solutions. [You do not have to solve the DE to do this, if you remember Thm 6.3].
3) (10pts) Choose ONE proof. Explain thoroughly.

a) Thm 9.8: \( L\{f \ast g\} = L\{f\}L\{g\} \).

b) Thm 9.9: \( L\{u_a(t)f(t-a)\} = e^{-as}F(s) \)

4) (15pt) Answer with True or False.

If \( f \) is a positive function, then \( L(f) \) is also positive.

If \( f \) is a continuous function, then \( L(f) \) is also continuous.

If \( f \) is periodic then \( L(f) \) is periodic.

The inverse LT of \( Y \) cannot have two different answers.

At an ordinary point \( x_0 \) of the usual DE, there is a power series solution that converges for some \( x \neq x_0 \).
5) (10pts, ex: 3.2.1) A stone weighing 4 lb falls from rest from a great height. As it falls, it is acted on by air resistance that is numerically equal to \( v/2 \), where \( v \) is velocity. Find the velocity and distance fallen at time \( t \).

Set up the appropriate initial value problem, but do not solve.

6) (10pts, ex: 9.5.1) Solve the system using the Laplace transform:

\[
\begin{align*}
x' + y &= 3e^{2t} \\
y' + x &= 0 \\
x(0) &= 2, \quad y(0) = 0
\end{align*}
\]
7) (10pts, Ex: 9.22) Find the inverse Laplace transform of the function below (small hints included, but you’ll need to find A):

\[
\frac{s^2 + 5s - 3}{(s + 2)(s - 1)^2} = \frac{A}{s + 2} + \frac{B}{s - 1} + \frac{C}{(s - 1)^2} \text{ where } A = ??, \quad B = 2, \quad C = 1
\]

8) (5pts, ex: 9.4.3) Find the Laplace transform of this piecewise-defined function: \( f(t) = 4 \) if \( 0 < t < 6 \) and \( f(t) = 0 \) if \( t > 6 \).