MAP 2302 May 5, 2016
Final Exam and Key Prof. S. Hudson

The questions are approx 10 points each unless labeled otherwise.

1) Transform $(3x + 2y + 3)dx + (6x + 4y - 1)dy = 0$ into a separable DE. Write out the new DE in standard form, but do not solve it.

2) Find the roots r_1 and r_2 of the indicial equation for the DE $x^2y'' + xy' + (x^2 - 4)y = 0$ and stop. As usual, imagine we are looking for a series solution on an interval $0 < x < R$, and notice that $x = 0$ is a regular singular point. Show all the work including several series (even if you might know the answer), but you do not have to solve the DE.

3) [7 pts] Transform the DE $x^3y''' + 4x^2y'' + xy' - 6y = -18x^2 + 1$ into one with constant coefficients and write out the characteristic equation for that, and stop. You do not have solve either DE.

4) [8 pts] Find $L(f)$ using the u_a method, where

$$f(t) = \begin{cases} 1 & \text{if } 0 < t < 2 \\ 2 & \text{if } 2 < t < 4 \\ 3 & \text{if } 4 < t < 6 \\ 0 & \text{if } t > 6 \end{cases}$$

5) Compute the inverse Laplace transform of this function. Hint: one of the partial fractions is $\frac{s-2}{s^2+4}$.

$$\frac{5s^2 - 8s + 20}{(2s - 4)(s^2 + 4)}$$

6) Start solving this system: $x' - 5x + 2y = 3e^{4t}$ and $y' - 4x + y = 0$, given that $y(0) = 0$ and that $x(0) = 3$. You can stop when you have found either $x(t)$ or $y(t)$. You do not have to compute both.

7) Use a Laplace transform to solve this IVP, where δ is the usual Dirac mass: $y' + y = \delta(t - 2)$ with $y(0) = 1$. Simplify completely, writing the solution in piecewise form if necessary. Do not worry if $y(t)$ is not continuous.

8) [15 pts, total] Short answers:

a) Suppose a simple harmonic motion (from Ch 5.2) has the formula $x(t) = 4\sin(\lambda t) + 3\cos(\lambda t)$. What is the amplitude of the motion ?

b) Give the standard form of a Bernoulli DE, from Ch. 2.3.B.

c) Give the formula for an integrating factor μ, for a linear DE $y' + P(x)y = Q(x)$.

9) [20pts] Answer True or False. You do not have to explain.

Every solution of $y'' + 4y = 0$ can be expressed as $y(t) = c_1 \cos(2t + c_2)$.

Every solution of $dx + dy = 0$ can be expressed as $y(x) = C$.

$L(\delta(t) + u_1(t) + e^t) = 1/(s-1) + e^{-s}/s + 1$ for $s > 1$.

The UC set of $x \sin(x)$ contains exactly four functions.

In Ch. 6.2, when $r_1 = r_2$, the general solution always includes a logarithm term.

A critically damped motion can pass through the equilibrium position multiple times.

One particular solution of $y'' + y = x$ is $y(x) = \sin(x) + x$.

Given that $y = e^{2x}$ solves $y' = 2y$, a second LI solution can be found by reduction of order.

The EU Thm in Ch.1 assumes that f_y is continuous.

Variation of parameters is normally used to solve linear homogeneous higher order DE's.

Reminders: Put your name on your exam (and initial any scratch paper). Reread the questions to check that you answered each one as best you can. Don't leave any TF blank. Have a good summer !

Remarks and Answers: The average was 67% with high scores of 98 and 94. The worst results were on problem 1 (38%) and best on problems 2 and 5 (both 87%). Problems very similar to 1 and 2 appeared on midterms. I wanted to see if people improved on those two topics - and got two very different answers.

I do not make a separate scale for final exams. It seems unlikely that this exam will change the semester scale on the Exam 3 Key much.

1) Set $z = 3x + 2y$ (see Ch.2.4). Get $(-2z + 9/2)dx + (z - 1/2)dz = 0$.

2) Since this is a Bessel DE, it is predictable that the roots are ± 2, but the instructions require showing the work. See Ch.6.2 and 6.3 for similar problems.

3) Use $x = e^t$ and $x^3 y''' = \frac{d^3y}{dt^3} - 3\frac{d^2y}{dt^2} + 2\frac{dy}{dt}$, etc, to get the CC DE

$$\frac{d^3y}{dt^3} + \frac{d^2y}{dt^2} + \frac{dy}{dt} - 6y = -18e^{2t} + 1$$

with char eqn $m^3 + m^2 + m - 6 = 0$.

4) $f(t) = 1 + u_2(t) + u_4(t) - 3u_6(t)$, so $F(s) = (1 + e^{-2s} + e^{-4s} - 3e^{-6s})/s$.

5) $\frac{3}{2}e^{2t} + \cos(t) - \sin(t)$.

6) $x(t) = 5e^{4t} - 2e^t$ and $y(t) = 4e^{4t} - 4e^t$. See HW 9.5.3. Most people started off OK and got at least half credit. The most common problems were algebraic. Many finished

correctly in the space provided, with efficient algebra and not much partial fraction work. Unfortunately, others got a bit lost and wrote 2 pages, usually with errors.

7) $y(t) = \begin{cases} e^{-t} & \text{if } 0 < t < 2 \\ e^{-t} + e^{-(t-2)} & \text{if } t > 2 \end{cases}$

8a) 5

8b) $y' + P(x)y = Q(x)y^n$

8c) $\mu = \exp(\int P \; dx)$

9) TFTTT FTFTF. The first one is True based on the trig identities used in Ch.5.2.