MAP 2302 July 1, 2013
Quiz I and Key Prof. S. Hudson

This should take about 20 minutes, with a lecture afterwards.

1) [40 pts] Solve the I.V.P: $\frac{\partial^2 y}{\partial x^2} - \frac{\partial y}{\partial x} - 12y = 0$, $y(0) = 2$, $y'(0) = 1$. Hint: the general solution to the DE is $c_1 e^{4x} + c_2 e^{-3x}$.

2) [30 pts] Label each DE with its type - exact, separable, homogeneous (as defined in Ch.2.2B), linear, Bernoulli (and not linear) or 'none of the above'.

a) $(3x^2 + 4y^2)dx + xydy = 0$

b) $\frac{dy}{dx} + \frac{3y}{x} = \frac{6x}{y^4}$

c) $(3x^2 + 4xy)dx + (2x^2 + 2y)dy = 0$

d) $\frac{dy}{dx} + \frac{3y}{x} = \cos(y)$

e) $x \sin y dx + x \cos y dy = 0$

f) $\frac{dy}{dx} + \frac{3y}{x} = 6x^2$

3) [30 pts] The linear DE $\frac{dx}{dt} + \frac{x}{t^2} = \frac{1}{t^2}$ has an integrating factor of the form $\mu(t)$.

a) Use a standard formula from Ch. 2.3 to find $\mu(t)$ (or, if you have forgotten it, you might be able to derive it as we did in class Friday).

b) Check that μ converts the linear DE into an exact DE. You do NOT have to solve it.

Remarks and Answers: The average among the top 25 was approx 75/100, which is good. There were two perfect scores. You can use the scale on the syllabus to assign yourself a letter grade, A's = 85-100, B's = 75-84, etc. This quiz counts approx 5% of your semester grade.

1) $y(x) = e^{4x} + e^{-3x}$. This was very similar to the HW from Ch.1.3. Use $y(0) = 2$ to get $c_1 + c_2 = 2$ and so on, to get $c_1 = c_2 = 1$. Most people got most of this work right, but some stopped at $c_1 = c_2 = 1$ and did not write out the answer, or wrote it incorrectly as $e^{4x} + e^{-3x} = C$, etc.

Suggestion: pay attention to the big picture (during the quiz, but also when studying). In this problem, you are given $y(x) = c_1 e^{4x} + c_2 e^{-3x}$. You need to compute the c_i, and plug them in. But the values of the c_i are not the actual answer. And this is not one of those problems from Ch.2 where you compute $F(x, y)$ and then set $F = C$.

2) Homog, Bern, Exact, None, Sep, Linear. Each part should have one correct answer. I mentioned before the Quiz that a DE such as $\frac{dy}{dx} + \frac{3y}{x} = 6x^2$ could (with some imagination) be called a Bernoulli equation, but to call it linear instead. Do not answer any parts with 'not linear', which is not one of the basic types.

3a) Since the variables are t, x instead of x, y we adjust the formula slightly to

$$\mu(t) = \exp(\int P(t)\ dt) = \exp(\int t^{-2}\ dt) = e^{-1/t}$$

3b) Multiplying by μ, the DE becomes $e^{-1/t}\frac{dx}{dt} + \frac{e^{-1/t}x}{t^2} = \frac{e^{-1/t}}{t^2}$ so that $M = e^{-1/t}$ and $N = \frac{e^{-1/t}(x-1)}{t^2}$. Both partials are $\frac{e^{-1/t}}{t^2}$ so the new DE is exact.

By a coincidence (which I did not notice until grading) this DE is also separable. If you noticed this at all, it was best to just ignore it. It does not help in following the given instructions.