MAP 2302 July 12, 2013

Quiz II and Key Prof. S. Hudson

This should take about 15 minutes (but it took 20-25), with a lecture afterwards.

1) [50 pts] Given that $y = (x+1)$ solves the DE $(x+1)^2 y'' - 3(x+1)y' + 3y = 0$, find a second L.I. solution.

2) [50 pts] Find the general solution using variation of parameters: $y'' + y = \tan^2(x)$.

Bonus [5 points]: Check that your answer to part (2) solves the D.E.

Remarks and Answers: The average was about 65 / 100 based on the top half, with high scores of 98, 95 and 85. This is a bit low, but in the normal range. The unofficial scale is

A's 77 to 100
B's 65 to 76
C's 55 to 64
D's 45 to 54

1) You are given one solution, and are expected to use reduction of order to find a second one. So, $y = v \cdot (x+1)$ leads to $(x+1)v'' - v' = 0$ or $(x+1)w' - w = 0$ which is separable. Get $|w| = C|x+1|$ or (safely ignoring absolute values and constants) just $w = x+1$, so $v = (x+1)^2$ (it's also OK to use $v = x^2/2 + x$, etc). So $y = (x+1)^3$. It is easy to check this is LI and is a solution.

The above explanation follows the method of Ex 4.16. A couple of people used the formula from Thm 4.7 Conclusion 2 instead, and got the same v.

You could treat this like a Cauchy-Euler DE, instead, and set $x + 1 = e^t$, etc.

There is a slight problem with the textbook here (see Conclusion 3 and answers to the exercises). The general solution is NOT $c_1(x+1)+c_2(x+1)^3$. For example, $y(x) = |x+1|^3$ is a solution to this problem that does not fit that pattern. This is probably not a very serious issue and may be understood by simply graphing the solutions.

2) $y_p = v_1(x)\sin(x) + v_2(x)\cos(x) = \sin(x)\ln|\sec x + \tan x| - 2$. You should get y_c easily from memory and add it in at the end. You can plug into the usual formulas to get

$$v_1 = -\int \frac{F\ y_2}{a_0\ W}\ dx = -\int \frac{\tan^2(x)\cos(x)}{-1}\ dx = \ln|\sec x + \tan x| - \sin(x)$$

$$v_2 = \int \frac{F\ y_1}{a_0\ W}\ dx = \int \frac{\tan^2(x)\sin(x)}{-1}\ dx = -\sec x - \cos x$$

If you got stuck here, you may need to practice integration or basic trig identities. This was exercise 4.4.2 and was done in class.

B) From $y = \sin x \ln|\sec x + \tan x| - 2$, get $y' = \sin x \sec x + \cos x \ln|\sec x + \tan x|$ and $y'' = \sec^2 x - \sin x \ln|\sec x + \tan x| + 1$. So $y'' + y = \sec^2 x - 1 = \tan^2 x$. Check!