

Here is a rough solution to problem 17 on page 68 (Ch.2.4)

We can assume that  $\mu$  and  $\nu$  are IF's. This means we can get two exact DE's from the given DE. We use the usual criteria for *exact* (and the Product Rule) to conclude that

$$\mu M_y + \mu_y M = \mu N_x + \mu_x N \quad (0.1)$$

$$\nu M_y + \nu_y M = \nu N_x + \nu_x N \quad (0.2)$$

where  $M_y$  is a common abbreviation for the partial derivative  $\partial M/\partial y$ .

Now, assume we have a function  $y(x)$  defined implicitly by the given formula,  $\mu(x, y(x)) = c\nu(x, y(x))$ , where  $c$  is a fixed constant from now on. Taking  $\partial/\partial x$  of both sides we get

$$\mu_x + \mu_y \frac{dy}{dx} = c\nu_x + c\nu_y \frac{dy}{dx}. \quad (0.3)$$

This is from multivariable calculus, but if you haven't had that course, you can get this from the formula for the total differential (see the lecture notes on  $dF$ ). Subtle remark: even though  $\mu = c\nu$ , it is probably NOT true that  $\mu_x = c\nu_x$  on the given curve. Now, multiply (0.2) by  $c$  and subtract that from (0.1). Using the formula  $\mu = c\nu$ , this simplifies to

$$[\mu_y - c\nu_y]M = [\mu_x - c\nu_x]N \quad (0.4)$$

Combining this with (0.3) we get

$$\frac{dy}{dx} = \frac{\mu_x - c\nu_x}{-\mu_y + c\nu_y} = \frac{-M}{N} \quad (0.5)$$

which implies  $Mdx + Ndy = 0$ , which means  $y(x)$  is a solution of the given DE.