

There have been several questions about Problem 13 from Ch 1.3, and the more I look at it, I must agree it is strange. Here is a similar example, with my solution and some comments. Given

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} 5 \\ 11 \end{pmatrix} \quad \mathbf{c} = \begin{pmatrix} -1 \\ -5 \end{pmatrix}$$

a) Write  $\mathbf{b}$  as a LC of  $\mathbf{a}_1$  and  $\mathbf{a}_2$ .

Solution: Set  $\mathbf{b} = x_1\mathbf{a}_1 + x_2\mathbf{a}_2$ . When you write this out, it means  $x_1 + 2x_2 = 5$  and  $3x_1 + 4x_2 = 11$ . We can solve this linear system and get  $x_1 = 1$  and  $x_2 = 2$ . So,  $\mathbf{b} = \mathbf{a}_1 + 2\mathbf{a}_2$  is the LC we want.

NOTE: Probably the author intended us to find  $x_1$  and  $x_2$  another way, perhaps by trial and error. I'm not sure. That is not usually easy to do, but with the numbers in problem 13 of the book it is not TOO hard.

b) Use the result from a) to determine a solution to  $Ax = b$ . Does it have any other solutions?

Solution:  $\mathbf{x} = [1 \ 2]^T$ . (It is a column vector, but it is easier to type as a row vector transposed). The system does not have other solutions, since it reduces to triangular form (work not shown).

c) Write  $\mathbf{c}$  as a LC of  $\mathbf{a}_1$  and  $\mathbf{a}_2$ .

Solution: This is similar to a), but now we get  $x_1 = -3$  and  $x_2 = 1$ .