Grading HW 3 and partial key - Feb 14, 2011 - S Hudson

Note from Feb 2, 2014: I don't recall 2011 very well, but apparently I was grading HW back then, at least briefly. We were using a different edition of our text then, so I have edited this page slightly for that. Probably the grading system is of little interest, but the answers below may be worthwhile.

15 points = did you try Ch 1.6 ? [I didn't look for mistakes on this one]
15 points = did you try the last three problems in Ch 3.2 ?
20 points = Ch. 2.2.15 graded carefully
20 points = Ch. 3.1.7 graded carefully
30 points = completeness, staple, etc

Comments, Answers:

2.2.15; Assume $AB = I$, with both $n \times n$. This does *not* imply $B = A^{-1}$ yet, because we don't know that $BA = I$. We need to prove that, and may need to prove A^{-1} exists first. Since we are in ch2, we may need some dets.

Proof: det $(A)\cdot$ det (B) = det (AB) = det $(I) = 1$. This shows det $(A) \neq 0$, which implies A^{-1} exists. So, we can use it on both sides; $A^{-1}AB = A^{-1}I$. This simplifies to $B = A^{-1}$. Done.

3.1.7; We need to show V can't contain two different vectors $\mathbf{v_1} \neq \mathbf{v_2}$, that both satisfy the definition of the zero vector; $\mathbf{x} + \mathbf{v_1} = \mathbf{x}$ and $\mathbf{x} + \mathbf{v_2} = \mathbf{x}$. But these equations imply $\mathbf{x} + \mathbf{v_1} = \mathbf{x} + \mathbf{v_2}$, and (using $-\mathbf{x}$) that $\mathbf{v_1} = \mathbf{v_2}$. So these two 'zero vectors' are actually the same.

More remarks on this: it is OK to use the notation $\mathbf{0}$ instead of $\mathbf{v_1}$ and maybe $\mathbf{0_2}$ instead of $\mathbf{v_2}$. Also, I was vague about what $\mathbf{x}$ stands for - I imagine it to be some other vector in V, but it isn't quite clear V has one! So, a slightly better proof would be; $\mathbf{v_1} = \mathbf{v_1} + \mathbf{v_2} = \mathbf{v_2}$ (explain these eqns!).