MAS 3105 Jan 20, 2011
Quiz I and Key Prof. S. Hudson

1) Use Gaussian elimination to put the following system into reduced row echelon form. Use matrix notation. You don't have to find the solution set.

$$\begin{aligned} x_2 + x_3 &= 0 \\ 3x_1 + 2x_2 + x_3 &= 4 \end{aligned}$$

2) Label each system as underdetermined, overdetermined or square. Then describe how many solutions there are (maybe infinity!), and explain that briefly.

$$A = \begin{pmatrix} 1 & 2 & | & 3 \\ 0 & 1 & | & 2 \\ 0 & 1 & | & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 & 0 & 1 & | & 5 \\ 0 & 0 & 1 & 3 & | & 4 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 & | & 5 \\ 0 & 1 & | & 4 \end{pmatrix}$$

3) Answer each part with "True" or "False". You don't have to explain (but it doesn't hurt, and might help if we decide later that a question was not totally clear).

a) A square matrix in REF must have a lead one somewhere in the top row.
b) A 3 by 4 matrix in RREF must have at least 9 zeroes.
c) A 4 by 3 matrix in RREF must have at least 9 zeroes.
d) Gaussian elimination can change an inconsistent system into a consistent one.
e) An undetermined homogeneous system must have at least two solutions.

Remarks and Answers: This was not supposed to be very hard, and the average [based on the top 14 out of 18 grades] was high; 51/60, or 85%. The unofficial scale is
A's = 55-60
B's = 49-54
C's = 43-48
D's = 37-42
F's = 0-36

1) Start by swapping the rows (a Type I op):

$$A = \begin{pmatrix} 1 & 0 & -1/3 & | & 4/3 \\ 0 & 1 & 1 & | & 0 \end{pmatrix}$$

2) A) Over; inconsistent ($x_2 = 1 = 2$ is not possible);
B) Under; infinitely-many solns (consistent with free variables);
C) Square, a unique solution, (5,4).

3) FFFFT. I usually go over the TF in class after each quiz. You are welcome to ask more about these though.