

1) Find the solution set, using  $\alpha$  notation, if necessary.

$$\begin{aligned}x_1 + 3x_2 + x_3 &= 2 \\x_2 - x_3 &= 4\end{aligned}$$

2a) Given the info below, is the system  $A\mathbf{x} = \mathbf{c}_1$  (where  $\mathbf{c}_1 = [11, 3]^T$ ) consistent? Explain.

$$A = \begin{pmatrix} -1 & 0 & 4 \\ 1 & 1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 4 \\ 2 & 0 \\ 3 & 7 \end{pmatrix}, \quad C = AB = \begin{pmatrix} 11 & 24 \\ 3 & 4 \end{pmatrix},$$

2b) Is  $\mathbf{c}_2$  a linear combination of  $\mathbf{a}_1$  and  $\mathbf{a}_3$ ? Prove your answer.

3) Give an example of a  $3 \times 3$  matrix  $U$  that is in RREF but is not the identity matrix, nor the zero matrix.

4) Answer each part with “True” or “False”.

a) In problem 1 above, there are 2 free variables.

b) None of the matrices A, B or C in problem 2 are symmetric.

c) A consistent underdetermined system has infinitely many solutions.

d) Gaussian elimination can convert any system to REF.

e) A system in RREF must be consistent.

**Remarks and Answers:** The best scores were 100, 98 and 97. The average score among the top 15 was 80. This may be the highest average among the quizzes this semester, but it is not too unusual for a Quiz 1. The other quizzes will probably focus more on HW-type problems and proofs, and the material gets a little harder for most people. The unofficial scale for Quiz 1 is

A's 87-100

B's 77-86

C's 67-76

D's 57-66

1)  $S = \{(-10 - 4\alpha, 4 + \alpha, \alpha)\}$ , from back substitution.

2a) Yes,  $A\mathbf{b}_1 = \mathbf{c}_1$  so  $\mathbf{x} = \mathbf{b}_1$  is a solution. I gave approx half credit for ‘Yes’ (do not forget to answer Yes or No!) and another half for a good explanation. That could take various forms, but it should include sound reasoning and enough words.

2b) Yes.  $4\mathbf{a}_1 + 7\mathbf{a}_3 = \mathbf{c}_2$ .

3) Replace the 3rd '1' in the identity matrix by '0', for example. There are many answers, but each should have either 1 or 2 '1's and the rest '0's.

4) FTTTF