MAS 3105 Sept 2, 2016
Quiz I Key Prof. S. Hudson

1) Use Gaussian elimination to put the following system into reduced row echelon form. Use matrix notation. You don't have to find the solution set. For a little extra credit, find the least number of GE steps required to do this and justify your answer.

$$\begin{aligned} x_1 + 2x_2 + x_3 &= 7 \\ 2x_3 &= 8 \end{aligned}$$

2) For each system A, B and C, decide i) if it is in REF, and ii) how many solutions there are (maybe infinity!), and explain briefly. So, 6 answers in total, plus explanations.

$$A) \begin{pmatrix} 1 & 2 & | & 3 \\ 0 & 1 & | & 2 \\ 0 & 1 & | & 1 \end{pmatrix}, \quad B) \begin{pmatrix} 1 & 2 & 0 & 1 & | & 5 \\ 0 & 0 & 1 & 3 & | & 4 \end{pmatrix}, \quad C) \begin{pmatrix} 1 & 1 & | & 5 \\ 0 & 1 & | & 4 \end{pmatrix}$$

3) Multiply the two matrices, if possible. Or, if not, explain why not.

$$\begin{pmatrix} 1 & 6 \\ 0 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 4 & 2 \\ 3 & 1 \\ 0 & 1 \end{pmatrix}$$

4) Answer each with "True" or "False". You don't have to explain.

a) A 3 by 3 square matrix in RREF must have 3 lead ones.
b) A 3 by 5 matrix in RREF must have at least 6 zeroes.
c) A system can have exactly three solutions.
d) The set of matrices $R^{2\times 2}$ is closed under addition.
e) An inconsistent system cannot be homogeneous.

Remarks: The problems were 15 points each, for a total of 60 points. If you like, you can convert to a 100 point system, so 45/60 becomes 75/100, etc (I will do that, in effect, at the end).

The average was approx 55 out of 60, based on the top 2/3's of the scores, which is very high (but not too unusual for Quiz 1). The unofficial scale below is higher than the one on the syllabus, but probably more informative. The future scales will almost certainly be lower. See the keys to my 2014 quizzes to see a typical pattern, if interested.

A's 55 to 63
B's 50 to 54
C's 45 to 49
D's 40 to 44

Answers: 1)

$$\begin{pmatrix} 1 & 2 & 1 & | & 7 \\ 0 & 0 & 2 & | & 8 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 1 & | & 7 \\ 0 & 0 & 1 & | & 4 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 0 & | & 3 \\ 0 & 0 & 1 & | & 4 \end{pmatrix}$$

So, it can be done in 2 steps. Not less, because changes are required in both row 1 and in row 2 (because the RREF is unique). One step cannot change two rows, unless it is a swap, which clearly would not get to RREF.

2) The REF answers were worth 2 points each and the others 3 points each (mostly for the explanations).

2A) Not REF. No solutions, since rows 2 and 3 imply $x_2 = 2$ and $x_2 = 1$ at the same time. [You could also perform a GE step to get it into REF and explain from there.]

2B) REF. Infinity solutions, since it is consistent with a free variable. I did not deduct if you failed to mention 'consistent'. That is somewhat important, but also fairly obvious.

2C) REF. One solution, since it is consistent with no free variables. [You could also explain this by finding the solution.]

3) We cannot multiply an $m \times n$ times a $3 \times r$ unless $n = 3$.

4) FTFTT